А мир устроен так,
Что всё возможно в нём,
Но после ничего
Исправить нельзя.

„Этот мир“, Л. Дербенев

Complementarity of the Deterministic Past and the Probabilistic Future as the Source of Nature Evolution

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Igor Furgel

(office@furgel.com)

Based on the postulates (i) of time-ensemble (discreteness of time), (ii) of least resources consumption and (iii) of discreteness of space, it is shown that the future and the past have fundamentally different characters: the future is probabilistic and the past – deterministic.

From these basic principles it was inferred that observable and existable states of nature can only be probabilistic, so that time progress is essentially vectored and irreversible. It is also shown that the entire evolution of nature is fundamentally irreversible.

Further, it is shown that all dynamic laws (incl. classical and quantum mechanics, both theories of relativity, electrodynamics) and statistical physics can be derived from these basic principles, whereby a concrete form of dynamic laws also depends on the notion of general properties of coordinate systems defined.

There are also some exciting additional results: the 'physical' sense of Euler's number and a deeper reason for the difference between the Shannon and the thermodynamic entropy became comprehendible.
It became possible to define the term 'information' both in concrete and abstract ways. It was also shown that the Heisenberg uncertainty relations reflect the condition of observability of states.

It was shown that formation of self-organised objects and their associations is rather a very probable way of evolution of nature.

This contribution addresses the circle of readers interested in questions of general physics, of microstructure of spacetime and of related philosophical aspects.

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Content

1 Ergodic Hypothesis and Structure of Time

1.1 Ergodic Hypothesis

The ergodic hypothesis in statistical mechanics consists in the *assumption* that the time average is equal to the average over ensembles.

It means that when one observes (measures) an observable of a statistical system long enough, one gets the same average value for this observable, as if one ascertains the average over an ensemble of all possible simultaneous states of the statistical system.

The ergodic hypothesis was originally introduced by Ludwig Boltzmann[1]. According to that, the state trajectory of a statistical system in the phase space will arbitrarily approximate each point of the phase space in a finite time.

It is neither a trivial question nor self-evident that averaging over time and averaging over ensemble shall lead to the same results. Acting on the assumption that this equality is essential, one can deduce the ergodic hypothesis from a more general postulate which will be established below.

1.2 Time-Ensemble Postulate: Time Progress Creates an Ensemble

Postulate P.1 (the Time-Ensemble Postulate):

Time does not progress continuously, but discretely (in time quanta), and each time quantum generates exactly one microstate of nature. This discrete time flow produces an ensemble of microstates.

This postulate transforms the ergodic hypothesis into one of its necessary consequences: the same averages of an observable at averaging over time and over ensemble represent neither a coincidence nor an incomprehensible property of nature, but are attributed to the time flow producing the ensemble. Hence, the average value over the time is being measured in fact over this ensemble. Averaging over time *is* ultimately averaging over the related ensemble; for this reason, the averages cannot possibly be unequal, which means that averaging over time and over ensemble does *not* cause *two different quantities* having identical values, but *one and the same quantity*.

One can wonder whether it makes any sense to replace the ergodic hypothesis by the time-ensemble postulate. As we will see below, this does make sense, because the time-ensemble postulate is capable of explaining many more phenomena than the ergodic hypothesis by itself. It means that the time-ensemble postulate has a more general character than the ergodic hypothesis; the latter is merely one of its consequences.

Now, let us consider the time-ensemble postulate and its properties in a more detailed way.

[1] and replaced later on by the more precise quasi-ergodic hypothesis by Paul Ehrenfest

1.3 Time Microstructure: Discreteness of 'Time Flow' and Existence of a Time Quant

The time-ensemble postulate implicates the following:

P.1-1: The time flow is not continuous, but discrete.

P.1-2: There is a minimal interval (quant) of time. That defines itself so that nature is and stays in *exactly one and the same* microstate within this elementary time interval: There are – by definition – no state transitions within a time quant.

The term 'microstate' here is seen as an instantaneous state of nature that is defined/described by the entirety of all current values of all attributes of all objects of nature.

Thus, time represents *the distance* between two different microstates of nature. The term 'time' does not exist outside of time quanta, for example, there is no time between the 'neighbouring' time quanta.

The assumption suggests that the value of time quant is the Planck time $t_p \equiv \sqrt{\frac{\hbar G}{c^5}} \approx 5,4 \cdot 10^{-44} s$; i.e. nature performs $t_p^{-1} \approx 1,85 \cdot 10^{43}$ time steps in a second.

P.1-3: The current time quant is 'the presence'; all previous time quanta represent 'the past', all forthcoming time quanta – 'the future'.

1.4 Inferring the Ergodic Hypothesis

The ergodic hypothesis can immediately be inferred from the properties P.1-1 – P.1-3:

Each presence (each time quant) generates exactly one microstate of nature, i.e. there are N microstates after N time steps, or, in other words, there is an ensemble of these microstates consisting of N elements (microstates). One can say that the 'time flow' generates this ensemble.

Therefore, the averaging over time represents in fact the averaging over the related ensemble having been generated by the time progress.

Remark 1: Regarding the terms used:
A *macrostate* of an ensemble of its *microstates* is the state of nature after N time steps, which can be observed/measured (with its measurable macroscopic observables like entropy, temperature, velocity, etc.).
The current microstate of an ensemble is its current element at the time step $j \leq N$.

We can infer from these considerations that the averaging over an ensemble generally represents a primary operation, irrespective of what discrete quantity – time or other kind of 'generators' – generates this ensemble[2].

[2] cf. the controversial discussion between Radu Balescu (= the averaging over ensembles is primary) and Yuri Klimontovich (= the averaging over the time is primary): Balescu was right.

2 The Principle of Least Resources Consumption

2.1 Observability of States as an Effect of their Probabilistic Character

2.1.1 Definition of the Term 'Indeterminacy'

The term '**indeterminacy of a state**'[3,4], which I would like to consider here, can be very useful for the description and comprehension of some interrelations.

Definition:

Let p_j be the probability of a microstate j. The indeterminacy of this microstate j is then

$$u_j \equiv -\ln p_j . \tag{2.1}$$

If a microstate definitely occurs ($p_j = 1$), then $u_j = 0$ (the indeterminacy of the microstate is = 0).

If a microstate cannot possibly occur ($p_j = 0$), then $u_j \to \infty$ (i.e. this state is absolutely indeterminate, because it can never occur).

2.1.2 Indeterminacy and Entropy

How do the terms 'indeterminacy' and 'entropy' relate to each other?

2.1.2.1 Shannon-Entropy

It can be useful to distinguish between *thermodynamic* indeterminacy and *information* indeterminacy in certain situations. Thermodynamic indeterminacy is defined by (2.1); information indeterminacy $\equiv -\log_2 p_i$.

According to this definition of information indeterminacy, the Shannon-Entropy needs to be considered as the weighted (average) indeterminacy of an ensemble of microstates[5]:

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[3] or of an event

[4] German: 'Unbestimmtheit eines Zustands'; Russian: 'неопределённость состояния'.

The term 'uncertainty' also used for this quantity is less appropriate from my point of view, because it suggests a cognitive component: something uncertain can be well-defined, but we do not have enough information about it.

Indeterminacy expresses that something is *fundamentally* not well-defined, irrespective of our being informed of it.

[5] One also refers to them as 'local' states

$$\mathcal{E}_N(\text{Shannon}) \equiv -\sum_{i=1}^{N} p_i \log_2 p_i \equiv \sum_{i=1}^{N} p_i u_i \tag{2.2}$$

(the indeterminacy u_i of state `i` occurs with a probability p_i and, hence, is also weighted by p_i).

Below, I will only use the thermodynamic indeterminacy (2.1), for which I will provide reasoning in sec. 2.2.3.

2.1.2.2 Microcanonical Partition Function

The microcanonical partition function[6] is defined as $\Omega_i \equiv p_i^{-1}$, whereby p_i is the probability of microstate `i`.

The local thermodynamic entropy is then

$$\mathcal{E}_i \equiv k_B \ln \Omega_i = k_B(-\ln p_i) = k_B u_i \,,$$

whereby k_B is the Boltzmann constant. Thus,

$$\mathcal{E}_i = k_B \cdot u_i \,. \tag{2.3}$$

2.1.2.3 Quantum Mechanical Entropy of a Macrostate

The quantum mechanical entropy for an ensemble of microstates can be calculated as

$$\mathcal{E} \equiv -k_B < \ln \rho >= -k_B Sp(\rho \ln \rho) = -k_B \sum_i p_i \ln p_i = k_B \sum_i p_i u_i \,,$$

whereby $\rho = \sum_i p_i \left|i\right\rangle\left\langle i\right|$ is the density operator[7].

Thus,

$$\mathcal{E} = k_B \sum_i p_i u_i = \sum_i p_i \mathcal{E}_i \tag{2.4}$$

for an ensemble of microstates.

The explanations in sec. 2.1.2.1, 2.1.2.2 and 2.1.2.3 show that the information (Shannon) as well as the thermodynamic entropy represent a measure of indeterminacy. This is merely a confirmation that the term ‘indeterminacy’ as defined here is meaningful.

Comparing (2.2) with (2.4), it becomes obvious that the information entropy (Shannon) and the thermodynamic entropy are equal in the ‘language’ of indeterminacy (except for the normalising Boltzmann constant k_B).

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[6] German: Zustandsumme

[7] The expectation value of an observable is given by $< A >= Sp(\rho \cdot A)$.

2.1.3 Indeterminacy and Action Quanta: Complementary Characters of the Past and the Future

In the stationary macrostates (where the observable macrostate of a system is maintained)

$$p_j(\text{ampl}) \sim e^{i\Delta\Phi_j}, \tag{2.5}$$

whereby p_j represents the probability amplitude of the transition from microstate `(j-1)` to microstate `j` (i.e. the probability amplitude of microstate `j`) and $\Delta\Phi_j \equiv \Phi_j - \Phi_{j-1}$ – the phase change at the transition from microstate `(j-1)` to microstate `j` (see e.g. (6.5) in annex A.2, sec. 6.2 or [6], chap. 17 "Symmetry and conservation laws").

The indeterminacy amplitude of microstate `j` is then $u_j(\text{ampl}) \equiv -\ln p_j(\text{ampl}) \sim -i\Delta\Phi_j = -i\frac{S_j}{\hbar}$, whereby S_j represents the action[8] needed by the system in order to transit from microstate `(j-1)` to microstate `j`.

Thus,

$$\frac{S_j}{\hbar} \sim iu_j(\text{ampl}), \tag{2.6}$$

whereby $\hbar \equiv \frac{h}{2\pi}$.

The ratio $\frac{S_j}{\hbar}$ represents the number of action quanta. It means that the result (2.6) shall be read as follows:

The number of the action quanta needed for creation of a microstate `j` *is proportional to the indeterminacy of this microstate.*

For creation of an absolutely improbable microstate ($p_j = 0,\ u_j \to +\infty$) nature would need an infinite number of action quanta ($\frac{S_j}{\hbar} \to \infty$)[9]; hence, such microstates[10] cannot exist.

For creation of a deterministic microstate ($p_j = 1,\ u_j = 0$) nature would require no one action quant ($\frac{S_j}{\hbar} = 0$)[11]. Such states, requiring no resources for their creation (and action does behave like a resource of nature; more about this in sec. 2.3.2 below), represent an empty set and, therefore, are (fundamentally!) not observable.

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[8] German: Wirkung

[9] If the amplitude of a quantity is infinite, the quantity itself is also infinite.

[10] and, of course, also the macrostates representing an ensemble of these microstates.

[11] If the amplitude of a quantity equals zero, the quantity itself also equals zero.

From the expression (2.6) follows for existable and observable states:

$$0 < u_j < \infty . \qquad (2.7)$$

The first inequality $0 < u_j$ assures observability, the second $u_j < \infty$ – existability of states.

From these considerations one can infer two reasons for the inexistence of a state:

i) this state requires infinitely many resources of nature and, hence, is unreachable or

ii) it represents an empty set and, thus, is unobservable.

Since – by definition – $p_j \equiv e^{-u_j}$ (s. (2.1)), one gets for the probability of an observable and existable state (from (2.7)):

$$0 < p_j < 1 . \qquad (2.8)$$

It means that observable and existable states cannot be deterministic; therefore, they must be probabilistic! The future is probabilistic.

In contrast, the **past** is definitely **deterministic**, because all decisions about any microstate have already been made there: everything in the past has already happened, the past represents a kind of event protocol of nature.

These terms are not directly applicable to the presence: the presence might represent a deterministic-probabilistic synthesis, because it probabilistically arises and deterministically resigns.

A direct consequence of the fundamentally different, complementary characters of the future and the past (probabilistic vs. deterministic) is that **time progress is vectored and irreversible** (s. sec. 2.6 on this subject).

It is principally impossible to probabilistically formulate the past and to deterministically arrange the future: what has already happened cannot possibly be probabilistic, because all decisions have already been taken; everything that can still happen, must be probabilistic in order to be observable and existable.

It means that a certain direction of time progress is neither a coincidence nor a fluctuation, but a necessary consequence of the different, complementary characters of the future and the past.

2.1.4 Other Sense of the Heisenberg Uncertainty Principle

In the light of these considerations, it is possible to newly interpret the Heisenberg uncertainty principle. Indeed,

$$\Delta t_j \Delta E_j \geq \hbar \;\Rightarrow\; \Delta S_j \geq \hbar \;\Rightarrow\; \frac{\Delta S_j}{\hbar} \geq 1 .$$

This form of the uncertainty relation states that nature has to spend at least one action quant in order to create a microstate. It is thoroughly commensurate with the conclusion in sec. 2.1.3 that the indeterminacy is $u_j > 0$ for observable states and, hence, $\frac{S_j}{\hbar} > 0$: Since $\frac{S_j}{\hbar}$ is the *number* of action quanta, the necessary condition of observability $\frac{S_j}{\hbar} > 0$ is equivalent to the condition $\frac{S_j}{\hbar} \geq 1$ [12].

It means that the Heisenberg uncertainty relations reflect the condition of observability of states and, in this way, their property to be probabilistic (cf. (2.8), $p_j < 1$).

2.1.5 The Principle of Least Resources Consumption: Least Action and Most Entropy

From (2.6), (2.7) and (2.8) we can infer that existable and observable states require a *finite* amount $\frac{S_j}{\hbar}$ of action quanta in order to be created (i.e. generally, a finite amount of resources, because action represents an integral resource of nature; read more about this in sec. 2.3.2).

Postulate P.2 (the Postulate of Least Resources Consumption):

Ensembles of microstates (i.e. macrostates) evolve in such a way that resources of nature required for this are consumed most economically (minimally).

Hamilton's principle of least action directly follows from this postulate: Since action represents an integral resource of nature (s. sec. 2.3.2 below), the ensembles of microstates, which are being created by the time flow (cf. sec. 1.2), evolve so that the action needed for the creation of related macrostates takes the least value.

The least action of a macrostate (i.e. of the related ensemble of microstates) corresponds neither to the most nor to the least value of local indeterminacy u_j (its extremes cannot exist at all, cf. (2.7)), but to an 'optimal' (in the sense of minimal consumption of resources) value of u_j.

What can such an optimal value of local indeterminacy be?

Let us consider the expression for dimensionless entropy (cf. (2.4) and (2.1))

$$\mathcal{E}_N = \sum_{j=1}^{N} p_j u_j \equiv \sum_j u_j e^{-u_j} \equiv -\sum_j p_j \ln p_j \,. \tag{2.9}$$

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[12] To make a state observable, nature has to spend more than 0, thus at least 1 action quant.

The value of the entropy depends on the distribution of microstates `j = 1` to `N` of the ensemble upon p_j (or u_j). It means that the optimal value resp. the optimal distribution upon p_j shall be of such a kind that the value $\mathcal{E}_N$ of the entropy is optimised.

The expression in (2.9) has only one optimum (extremum), namely a maximum ($\mathcal{E} = \ln Z$), and this maximum will be achieved if the distribution of microstates is equiprobable (s. annex A.1 in sec. 6.1):

$$\mathcal{E}(thermodynamic) \approx \ln Z - \frac{1}{2}\sigma , \tag{2.10}$$

whereby σ represents the root-mean-square deviation of the local probability p_j from its equiprobable average and Z – the number of possible choices.

Summarised we can assert that the equiprobable[13] distribution of microstates of an ensemble (i.e. of a macrostate) maximises the entropy of the macrostate and minimises the action (and, thus, the resource) having to be spent.

From these considerations we can infer **the principle of most entropy**: An ensemble of microstates evolves in such a way that the entropy of the related macrostate is being maximised.

The principle of most entropy and the principle of least action are equivalent to each other and can be derived from the postulate of least resources consumption.

From the principle of most entropy one can directly infer that the related entropy production (the average entropy changing at each time step, $P_{\mathcal{E}}$) is also maximal. Indeed, let us assume that the entropy production $P_{\mathcal{E}j}$ at each time step `j` from `1` to `N` would not take the most possible value. Then, the entropy of the related macrostate $\mathcal{E}_N = \sum_{j=1}^{N} P_{\mathcal{E}j}$ would not also be maximised. It means that the entropy production $P_{\mathcal{E}}$ at each time step must take its maximally possible value in order to fulfil the principle of most entropy.

The principle of most entropy means that the number of opportunities (of microstates) for reaching the final macrostate grows as quickly as possible[14]. Therefore, one can pictorially say that nature evolves on the path of least resistance which in turn means minimising consumed resources.

The equivalence of the principle of most entropy and Hamilton's principle also means that the latter does not represent just a mathematical trick, but is a result of the principle of least resources consumption and of observability and existability of states, i.e. a result of the complementary characters of the past and the future (cf. sec. 2.1.3).

[13] The term 'equiprobable distribution' should here be understood as a distribution which ensures that the root-mean-square deviation σ in (2.10) is negligible ($\sigma \ll \ln Z$). This includes the strict equiprobable distribution as well as a set of other distributions around the strict equiprobable distribution.

[14] Maximum of entropy production

Even more: the principle of most entropy says that an ensemble of microstates evolves in such a way that the entropy of related macrostate is being maximised. It means that nature is evolving in such a way that it is producing the most possible entropy. On the other hand, the objects of nature producing maximal entropy are self-organised. Thus, we can infer from this **that formation of self-organised objects and their associations[15] is rather a very probable way of evolution of nature**.

Just as each *conservation law* is a result of a *symmetry* (Noether theorem), the principle of most entropy determining the manner of *changing* a state (i.e. of *evolution*) is, amongst other, a result of the *asymmetry/complementarity* of characters of the deterministic past and the probabilistic future.

It seems to be an interesting observation from a philosophical point of view: **Symmetry relates to conservation, asymmetry – complementarily – to evolution**.

2.2 Probabilistic Evolution of Nature: Information, Microstates, Alternatives and the Sense of Euler's Number

2.2.1 Information Value of a Macrostate

Let us ask ourselves now what relationship exists between the two neighbouring microstates: between the presence and the first future (= the presence + 1st time step).

The future is probabilistic (s. sec. 2.1.3 above). The *most probable* 'first future' is such that the postulate of least resources consumption is fulfilled on the macrostate (i.e. on the ensemble consisting of N microstates).

The principle of most entropy having been inferred from this postulate enforces that the entropy of the microstate after N time steps $\mathcal{E}_{j=N}$ is approaching its maximal value $\mathcal{E}_{\max}$ in the quickest way (i.e. in the minimal number of time steps[16]). Therefore, the information value of each next macrostate ($IV_{j=N} \equiv \mathcal{E}_{\max} - \mathcal{E}_{j=N}$) is also decreasing in the most rapid way. In this case, the system is passing into its 'most symmetric' state also in the quickest way.

When a system has reached in its 'most symmetric' macrostate (where the information value of the macrostate is $IV_{j=N_{\max}} = 0$), this macrostate cannot change any more, because each next macrostate would be identical to the previous. Hence – according to the definition of the term 'time' in P.1-2 in sec. 1.3 as the distance between two different microstates of nature – time progress for such a system will stop. Also, the information flow out of this system will stop at this moment, because $IV_{j=N_{\max}} = 0$.

This means that, when a system has reached its 'most symmetric' macrostate,

1) the term 'time' will not exist any longer within this system: the system will 'know' only the present; the system will have forgot the past; there will be no future for the system;

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[15] Biological objects also belong to the class of self-organised objects; their associations represent societies.

[16] Maximum of entropy production

2) each interaction with this system will be impossible due to absence of any information flow out of the system. It means that this system will not be observable and, thus, in the state of inexistence.

The information value of each next macrostate $IV_{j=N} \equiv \mathcal{E}_{\max} - \mathcal{E}_{j=N}$ after N time steps can be represented as follows, s. (2.4):

$$IV_{j=N} \equiv \mathcal{E}_{\max} - \mathcal{E}_{j=N} = k_B \left(\sum_{j=1}^{N\max} p_j u_j - \sum_{j=1}^{N} p_j u_j \right) = k_B \sum_{j=N+1}^{N\max} p_j u_j = \mathcal{E}_{N+1}^{N\max}, \qquad (2.11)$$

whereby $\mathcal{E}_{N+1}^{N\max}$ represents the entropy of all microstates of the system – starting with the time step N+1 and ending with the time step N_{max}, at which the information value of the macrostate is $IV_{j=N_{\max}} = 0$.

It means that the ‘remaining’ information value of a system at the time step N is the total entropy of all still in the future lying microstates of the system.

In other words, **the ‘remaining’ information value of a system at the time step N represents the weighted (average) indeterminacy of the completed future ensemble of microstates of the system**, cf. sec. 2.1.2.

The statement (2.11) enables giving a coherent definition for the term ‘information’, whereby the term ‘indeterminacy’ is introduced in (2.1), s. sec. 2.1.1:

Information is alteration of the degree of indeterminacy.

Information provides matter with the *form* of its existence: information makes the matter structured, inhomogeneous, and, in this way, alters (diminishes) its degree of indeterminacy; sec. 2.4 treats the relation between matter and information in greater detail.

If a system loses information – by system-internal processes and by its interaction with the outer world –, its degree of indeterminacy alters (increases). When the indeterminacy of a system reaches its maximal value, no alteration of this quantity and, hence, also no information flow out of the system is possible any longer.

2.2.2 Alternatives: The Fundamental Approach

Let us consider now the single time steps being rendered by nature during its evolution in a more detailed way, i.e. we will now consider each single microstate of an ensemble starting at the current state of the present.

Since the future of nature is probabilistic, the latter has to choose a certain ‘path’ from the possible alternatives, and do that in a way that the entropy of the ensemble is maximal (s. sec. 2.1.5).

The alternatives can be binary, trinary, tetrary and so on. Let us term this property '*Dimension of an Alternative*' α, whereby $\alpha \geq 2$ [17].

The following figure illustrates this idea:

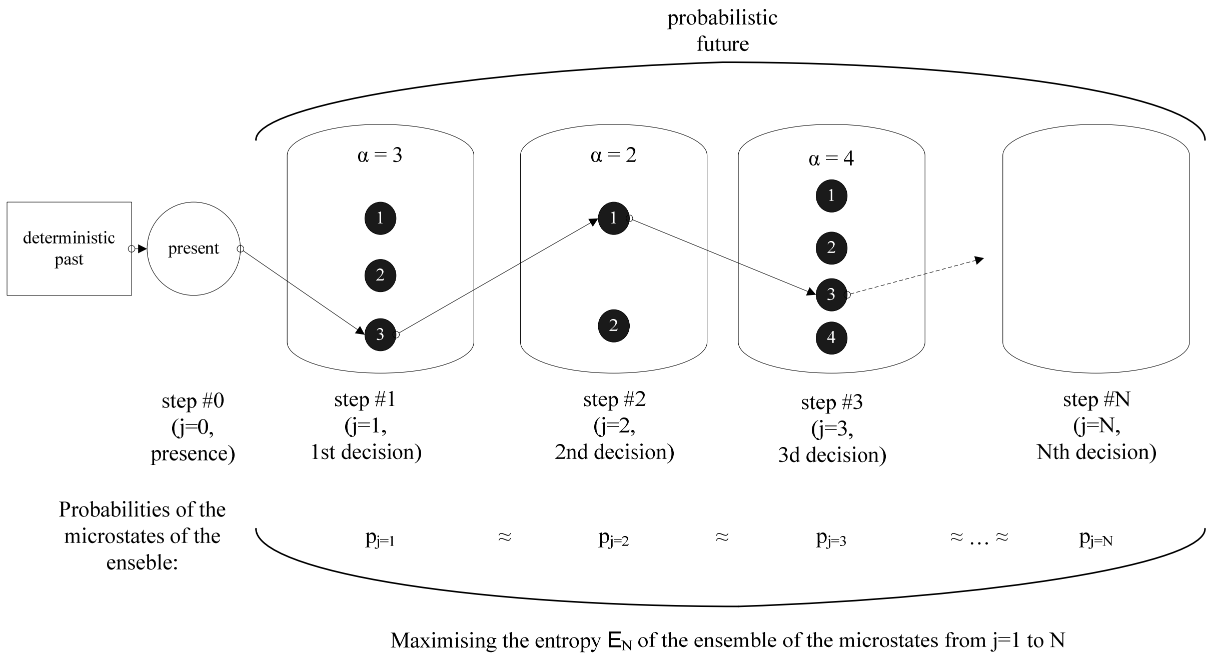

Figure 1: Alternatives within the microstates of an ensemble

At this point we should differentiate further types of states:

a) The *microstate of an alternative* is the sequence of the decision nodes within the alternative (e.g. (1)->(2)->(3) as shown in Figure 1, the alternative at the step #1). There can be $\alpha!$ of such sequences;

b) The (decided) *macrostate of an alternative* is that any one of α possible decisions has been made within the alternative.
The succession of microstates of an alternative is not important here: if a certain microstate of the alternative (e.g. #(1) of the alternative at step #2 as shown in Figure 1) enables maximising the entropy of the ensemble, this microstate #(1) will be chosen by nature irrespective of the position in the sequence where this microstate #(1) stands.

c) The current *microstate* j *of an ensemble* has already been defined in sec. 1.4 (s. Remark 1 there). It exactly depicts the macrostate of the just decided alternative and is represented by the current time step $j \leq N$.

d) The *macrostate of an ensemble* of its microstates is the state of nature after N time steps that can be observed/measured (with its macroscopic observables like entropy, temperature, etc.), cf. the definition in sec. 1.4, Remark 1.

[17] $\alpha = 1$ would mean that it is no alternative, but a deterministic conditioned decision.

2.2.3 Distribution of Dimensions of Alternatives. The Sense of Euler's Number

Alternatives can be binary, trinary, tetrary and so on. It means that the particular dimension α of different alternatives can take a value in the range $2 \le \alpha < \infty$.

This fact raises the question of the distribution of dimensions of alternatives. We will now look at this question.

The partition function $\Omega(\alpha)$ for an alternative of the dimension α is the number of the microstates of the alternative, which instantiate its given macrostate. There are still $(\alpha - 1)$ macrostates within an already decided alternative[18], so that

$$\Omega(\alpha) = \frac{\alpha ![microstates]}{(\alpha - 1)[macrostates]}.$$

This leads to the probability of a macrostate of an alternative ($\Omega_i \equiv p_i^{-1}$):

$$\rho(\alpha) \equiv \Omega^{-1}(\alpha) = \frac{\alpha - 1}{\alpha !}.$$

Thus, the existence probability of an alternative of the dimension α is

$$\rho(\alpha) = \frac{\alpha - 1}{\alpha !}. \tag{2.12}$$

We now examine whether $\rho(\alpha)$ can indeed express probabilities: The sum of all possible values of $\rho(\alpha)$ must equal 1.

If one takes into account that

$$e \equiv \sum_{k=0}^{\infty} \frac{1}{k!} \equiv 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \equiv 2 + \sum_{k=2}^{\infty} \frac{1}{k!} \;\Rightarrow\; \sum_{k=1}^{\infty} \frac{1}{k!} \equiv e - 1;\; \sum_{k=2}^{\infty} \frac{1}{k!} \equiv e - 2,$$

one gets the following for the wanted sum

$$\begin{aligned} &\sum_{\alpha=2}^{\infty} \rho(\alpha) = \sum_{\alpha=2}^{\infty} \frac{\alpha - 1}{\alpha !} = \sum_{\alpha=2}^{\infty} \frac{\alpha}{\alpha !} - \sum_{\alpha=2}^{\infty} \frac{1}{\alpha !} = \sum_{\alpha=2}^{\infty} \frac{1}{(\alpha - 1)!} - \sum_{\alpha=2}^{\infty} \frac{1}{\alpha !} \to [\alpha - 1 \equiv \beta] \to \\ &\to \sum_{\beta=1}^{\infty} \frac{1}{\beta !} - \sum_{\alpha=2}^{\infty} \frac{1}{\alpha !} = (e-1) - (e-2) = 1. \end{aligned}$$

Finally,

$$\sum_{\alpha=2}^{\infty} \rho(\alpha) = 1, \tag{2.13}$$

which means that $\rho(\alpha)$ can indeed express probabilities.

The behaviour of the distribution (2.12) is depictured in the following figure:

[18] which represents its macrostate, s. the definition, item b) in sec. 2.2.2

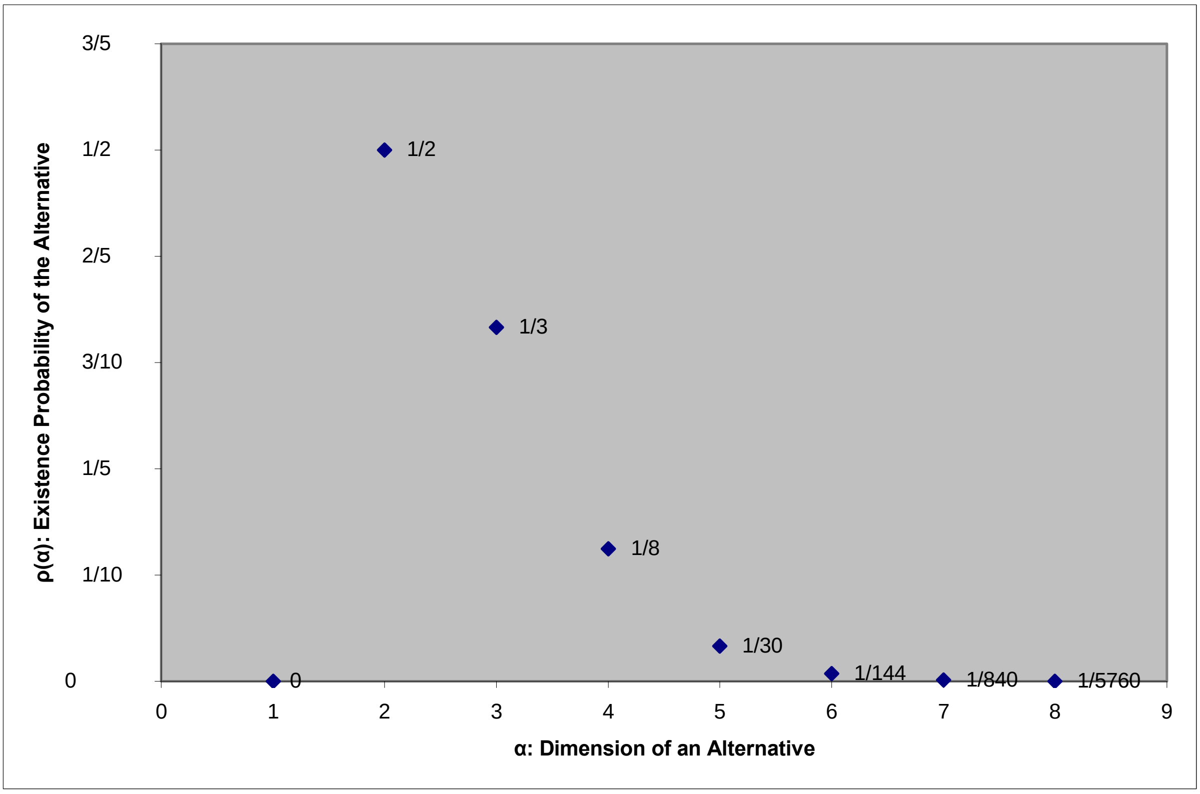

Figure 2: Distribution of dimensions of alternatives

This distribution $\rho(\alpha)$ shows that most alternatives are binary. Their dimension $\alpha = 2$ and the existence probability lies at $\frac{1}{2}$. Trinary ($\alpha = 3,\ \rho(\alpha) = \frac{1}{3}$) and tetrary ($\alpha = 4,\ \rho(\alpha) = \frac{1}{8}$) alternatives are also often encountered.

It is interesting that $\rho(\alpha = 1) = 0$. It means that the existence probability of an alternative of the dimension $\alpha = 1$ equals zero: **such a deterministic 'alternative' cannot exist in nature at all! This confirms the conclusion in sec. 2.1.3 that the future is probabilistic**.

Thus, we found the distribution of dimensions of alternatives. Now, we wonder what the average dimension of all existing alternatives with $2 \le \alpha < \infty$ is:

$$\overline{\alpha} \equiv \sum_{\alpha=2}^{\infty} \rho(\alpha) \cdot \alpha = \sum_{\alpha=2}^{\infty} \frac{\alpha - 1}{\alpha!} \cdot \alpha = \sum_{\alpha=2}^{\infty} \frac{1}{(\alpha - 2)!} \to [\alpha - 2 \equiv \beta] \to \sum_{\beta=0}^{\infty} \frac{1}{\beta!} = e \text{ (Euler's number).}$$

The average dimension of all existing alternatives is just Euler's number:

$$\overline{\alpha} = e. \qquad (2.14)$$

This result can be interpreted as the 'physical' sense of Euler's number.

Now, a deeper reason for the difference between the information entropy (Shannon) and the thermodynamic entropy is comprehendible (s. sec. 2.1.2.1):

- the information entropy (Shannon) is defined on the array of exclusively binary ($\alpha = 2$) alternatives;
- the thermodynamic entropy is defined on the array of all alternatives existing in nature ($2 \le \alpha < \infty$) with their 'natural' distribution $\rho(\alpha)$, s. (2.12). This 'natural' distribution (2.12) causes the average value of dimensions of alternatives $\overline{\alpha} = e$.

The consideration above made me assume that the approach of discrete Markov chains may be well appropriate to mathematically describe this kind of nature evolution – a discrete progress of time by decision of alternatives. However, this possibility will not be pursued in this contribution.

2.3 *Choice and Action. The Principle of Least Resources Consumption*

2.3.1 Indeterminacy Operator and Evolution of Nature

Having started in a macrostate A of an ensemble, nature will attain a macrostate B after N_{AB} time steps. On the path from A to B, nature has to 'analyse' a certain amount of options Z_{AB} in N_{AB} time steps as described in sec. 2.2.2. This number of options to be analysed can be calculated as $Z_{AB} = \alpha(j=1) \cdot \alpha(j=2) \cdot \ldots \cdot \alpha(j = N_{AB})$. It means that $Z_{AB} \approx \overline{\alpha}^{N_{AB}}$, if N_{AB} is sufficiently large, cf. Figure 1 in sec. 2.2.2; $\overline{\alpha}$ is the average dimension of all existing alternatives, s. sec. 2.2.3.

Likewise, if j represents the current macrostate of an alternative resp. the current microstate of an ensemble, then $\overline{\alpha}^j$ is the approximate amount of the options analysed heretofore (the amount of 'paths' to decide), if j is sufficiently large.

Then we can write down:

$$\frac{d}{dj}(\overline{\alpha}^j) = \ln \overline{\alpha} \cdot \overline{\alpha}^j = -\ln \overline{\alpha}^{-1} \cdot \overline{\alpha}^j = u_{\overline{\alpha}} \cdot \overline{\alpha}^j .$$

We here took into account that $\overline{\alpha}^{-1}$ represents the average probability of a choice within an alternative, i.e. the average probability that a certain node was chosen within an alternative, as well as the definition of indeterminacy (2.1) in sec. 2.1.1.

Thus,

$$\frac{d}{dj}(\overline{\alpha}^j) = u_{\overline{\alpha}} \cdot \overline{\alpha}^j . \tag{2.15}$$

The quantity `j` represents the space of decisions (actions[19]) in this formula: each time step `j` is making a decision on the imminent alternative; $u_{\bar{\alpha}}$ represents the average indeterminacy of a choice within an alternative.

That means that the indeterminacy operator $u_{\bar{\alpha}}$ in (2.15) generates the progress of nature in the space of decisions (actions).

We can see here a concrete instantiation of a general philosophical principle: ‘choice’ and ‘action’ represent an antinomy; they are complementary to each other.

Since $\overline{\alpha} = e$ (s. (2.14) in sec. 2.2.3), the average eigenvalue of the indeterminacy operator is

$$u_{\bar{\alpha}} \equiv -\ln \overline{\alpha}^{-1} = \ln e = 1 . \tag{2.16}$$

It means that the average local entropy $\mathcal{E}_j$ of every single time step is 1 bit (s. sec. 2.1.2.2). In other words, **a decision of each alternative entails on average 1 bit entropy**.

On the ‘path’ from macrostate A to macrostate B of the ensemble in N_{AB} time steps, nature has to decide exactly N_{AB} alternatives: it cannot skip any *decision*, because there are no alternatives with dimension $\alpha = 1$, s. sec. 2.2.3. It means that nature *has to decide* at each time step. If one assumes that nature manages/maintains only <u>one</u> state space (as postulated in P.1), one directly reasons that **nature generates on average 1 bit entropy per time step**[20].

Since each decision of nature increases its entropy on average by 1 bit, the entropy of the ensemble at the transition from A to B is

$$\mathcal{E}_{AB} \approx N_{AB} . \tag{2.17}$$

The entropy of an ensemble increases on average by 1 bit per time step. Accordingly, the information value $IV_{j=N}$ of the macrostate of the ensemble decreases on average by 1 bit per time step (s. sec. 2.2.2).

This result can also be inferred from other considerations, namely from the consideration of microstates of an ensemble *separately* – in the future and in the past. As already stated in this section, nature has to ‘analyse’ $\mathrm{Z}_{AB} = \alpha(j=1)\cdot\alpha(j=2)\cdot\ldots\cdot\alpha(j=N_{AB}) \approx \overline{\alpha}^{N_{AB}} = e^{N_{AB}}$ options in N_{AB} time steps for the transition from A to B. One can imagine that macrostate A is connected with macrostate B by $e^{N_{AB}}$ theoretically possible paths in the state space.

a) Future (before making any decision)
   Before making any of N_{AB} decisions (i.e. in macrostate A of the ensemble, before nature has ‘analysed’ and decided the first alternative), all macrostates

[19] German: Handlungen

[20] If one assumed nature would maintain, for example, two state spaces, it would have to decide twice at each time step and thus produce more entropy per time step (on average 2 bits).

of all alternatives are (yet) equiprobable (the principle of equal a priori probabilities), namely approximately $\bar{\alpha}^{-1} = e^{-1}$ (s. (2.14) in sec. 2.2.3). Since each theoretically possible path in the state space from A to B has N_{AB} alternatives, the theoretical probability to reach B using any one such a path is $e^{-N_{AB}}$. Since all theoretically possible paths in the state space are equiprobable, the entropy to be produced at the transition A->B is $\mathcal{E}_{AB} = \ln e^{N_{AB}} = N_{AB}$ (s. (6.1) in sec. 6.1).

b) Past (after having made all decisions)
After having made all N_{AB} decisions (i.e. in macrostate B of the ensemble, after nature has 'analysed' and decided the last alternative), the macrostates of all alternatives are not equiprobable any more: each alternative was unambiguously decided (according to P.2, s. sec. 2.1.5), so that the realised, decided macrostates of the alternatives became true ($p_j(decided) = 1$) and all remained, not realised macrostates of the alternatives became false. Herewith only one of $e^{N_{AB}}$ theoretically possible paths in the state space between A and B was realised. In other words, each alternative will most probably be decided in such a way that the entropy of the ensemble will be maximised (s. sec. 2.1.5), i.e. the decision made for an alternative is a very probable one.

These considerations allow a short formulation for complementary characters of the past and the future:

An act of decision-making conveys and converts the future into the past.

This allows reasoning the time irreversibility from another aspect: revising an already made decision represents itself a decision and conveys further on the future into the past; i.e. revising an already made decision cannot reverse time, s. sec. 2.6 on this subject.

2.3.2 Complementary Terms as Resource and the Principle of Least Resources Consumption

Let us establish here the following hypothesis:

Hypothesis H.1 (complementary terms as resource):
The product of complementary terms always acts/behaves like a 'resource'.

In order to illustrate this thought, let us consider two complementary terms:

(i) choice and

(ii) action.

As already indicated in sec. 2.3.1 (formula (2.15)), the time steps `j` on the 'way' from macrostate A to macrostate B of an ensemble represent the space of decisions (actions), whereby each time step `j` is a making of a decision on the forthcoming alternative, whose average dimension is $\overline{\alpha}$ [21] (choice within an alternative).

Then, the related choice-action resource is (action * choice)

$$R_{AB} \equiv N_{AB} \cdot \overline{\alpha}\,, \tag{2.18}$$

whereby N_{AB} is the amount of time steps `j` necessary in order to get from macrostate A to macrostate B.

The question now is: which average dimension $\overline{\alpha}$ minimises the resource R_{AB}?

The resource $R_{AB}(\overline{\alpha})$ as function of $\overline{\alpha}$ is

$$R_{AB}(\overline{\alpha}) \equiv N_{AB} \cdot \overline{\alpha} \approx \frac{\ln Z_{AB}}{\ln \overline{\alpha}} \cdot \overline{\alpha}\,, \tag{2.19}$$

whereby it was regarded that $N_{AB} \approx \frac{\ln Z_{AB}}{\ln \overline{\alpha}}$, s. sec. 2.3.1.

The first derivation is then $R'_{AB}(\overline{\alpha}) \approx \ln Z_{AB} \cdot \frac{\ln \overline{\alpha} - 1}{\ln^2 \overline{\alpha}}$.

The extremum of $R_{AB}(\overline{\alpha})$:

$R'_{AB}(\overline{\alpha}_{\min}) = 0 \;\Rightarrow\; \ln \overline{\alpha}_{\min} - 1 = 0 \;\Rightarrow\; \overline{\alpha}_{\min} = e$,

whereby it was regarded that the number of options Z_{AB} having to be 'analysed' by nature in N_{AB} time steps is greater than 1 (i.e. $\ln Z_{AB} > 0$)[22] and $\overline{\alpha} < \infty$ [23].

Thus,

$$\overline{\alpha}_{\min} = e\,. \tag{2.20}$$

It is obvious (cf. the behaviour of $R'_{AB}(\overline{\alpha})$) that $\overline{\alpha}_{\min} = e$ indeed is the only minimum of $R_{AB}(\overline{\alpha})$.

This shows that the least resource $R_{AB}(\overline{\alpha})$ will be consumed if the average dimension of alternatives is $\overline{\alpha} = e$. And that is indeed the case (s. (2.14) in sec. 2.2.3)**!**

Since $\overline{\alpha} = e$, the resource $R_{AB}(\overline{\alpha})$, as it is defined in (2.18), will be consumed in the most minimal/economical way during the transition from macrostate A to macrostate B of the ensemble.

---

[21] cf. (2.14)

[22] Indeed, $Z_{AB} = \alpha(j=1) \cdot \alpha(j=2) \cdot \ldots \cdot \alpha(j = N_{AB}) > 1$ due to observability of state ($u_{\overline{\alpha}} \equiv -\ln \overline{\alpha}^{-1} > 0$, cf. (2.7)) and to the probabilistic future, s. sec. 2.2.

[23] $u_{\overline{\alpha}} \equiv -\ln \overline{\alpha}^{-1} < \infty$ due to existability of state, cf. (2.7).

This behaviour of resource $R_{AB}(\overline{\alpha})$ is exactly commensurate with the postulate P.2 (sec. 2.1.5). Thus, one can conclude that the hypothesis H.1 as established above is at least plausible.

If one re-defines the hypothesis H.1 as a *postulate*, it is possible to derive from H.1 the postulate of least resources consumption P.2[24]. Hereby, we have to regard the relation (2.14) being entailed by the 'natural' distribution (2.12). In such a constellation, the following logical chain would be effective:

{the hypothesis on complementary terms as resources (treated here as H.1) + the 'natural' distribution of dimensions of alternatives (2.12)} =>

=> {the principle of least resources consumption (treated here as P.2)} =>

=> {Hamilton's principle of least action (and the equivalent principle of most entropy)}.

Let us now continue with the analysis of the choice-action resource (2.18). The optimised value of this function at $\overline{\alpha} = \overline{\alpha}_{\min} = e$ is

$$R_{AB}^{opt} \equiv R_{AB}(\overline{\alpha} = \overline{\alpha}_{\min}) = N_{AB} \cdot \overline{\alpha}_{\min} = e \cdot N_{AB} \approx e \cdot \mathcal{E}_{AB},$$

whereby (2.17) was regarded. $\mathcal{E}_{AB}$ is the entropy of ensemble of the microstates $j = 1 \div N_{AB}$ having been generated by the time progress during the transition from macrostate A to macrostate B of the ensemble. According to the principle of maximal entropy, the latter $\mathcal{E}_{AB}$ takes the maximally possible value, s. sec. 2.1.5.

Thus,

$$R_{AB}^{opt} \approx e \cdot \mathcal{E}_{AB}^{\max}. \qquad (2.21)$$

This relationship allows some interesting interpretations.

1) The entropy of an ensemble at the transition A -> B can be represented using the related indeterminacy as follows (s. (2.4); the entropy is here a dimensionless quantity, i.e. without the coefficient k_B):

$$\mathcal{E}_{AB} = \sum_{j=1}^{N_{AB}} p_j u_j \approx e^{-1} \sum_{j=1}^{N_{AB}} u_j = e^{-1} u_{AB}, \qquad (2.22)$$

whereby it was regarded that $p_j \approx \overline{\alpha}^{-1} = e^{-1}$ (s. sec. 2.3.1, item a)); indeterminacy, like entropy, is an additive quantity (from the definition in sec. 2.1.1).

If one combines (2.21) with (2.22), it yields

$$R_{AB}^{opt} \approx u_{AB} \qquad (2.23)$$

for the resource.

[24] Hamilton's principle of least action is also a deduction from P.2, s. sec. 2.1.5.

It means, if the indeterminacy were $u_{AB} = 0$ (and it would be the case if $\overline{\alpha} = 1 \neq e$, i.e. if the future were deterministic, s. sec. 2.2.3), the related resource would also be $R_{AB}^{opt} = 0$; i.e. nature would need no resources in order to create macrostate B. Such a macrostate would not be observable (s. sec. 2.1.3).

If the indeterminacy is $u_{AB} \to \infty$, the related resource would also be $R_{AB}^{opt} \to \infty$; i.e. nature would consume infinitely many resources in order to create macrostate B. Such a macrostate would not be able to exist (s. sec. 2.1.3).

2) The number of action quanta $\frac{S_{AB}}{\hbar}$ is proportional to the indeterminacy, as can be seen from (2.6). Then $\frac{S_{AB}}{\hbar} \sim iu_{AB}(\text{ampl}) \approx iR_{AB}^{opt}(\text{ampl})$, hence

$$\frac{S_{AB}}{\hbar} \sim iR_{AB}^{opt}(\text{ampl}) = i \cdot e \cdot \mathcal{E}_{AB}^{\max}(\text{ampl}) . \tag{2.24}$$

It means that the amount of action quanta for creation of macrostate B (which is equivalent to the creation of an ensemble of the microstates $j = 1 \div N_{AB}$) is proportional

- to the minimally necessary resource (s. also sec. 2.1.3 on this) or
- to the maximally possible growth of entropy.

3) Supposing nature would transit from A to B *not* in such a way that the resource R_{AB} required for this is minimal, then the entropy of nature would increase more than its maximal value $\mathcal{E}_{AB}^{\max}$, which would be a contradiction:

if $R_{AB}^{real} > R_{AB}^{opt} \;\Rightarrow\; \mathcal{E}_{AB}^{real}[= \frac{R_{AB}^{real}}{e}] > \mathcal{E}_{AB}^{\max}$; however, this relation can only be: $\mathcal{E}_{AB}^{real} \leq \mathcal{E}_{AB}^{\max}$.

Supposing now, transition A->B would happen in such a way that the real growth of entropy is *less* than its maximal value $\mathcal{E}_{AB}^{\max}$, then the really consumed resource for this transition would be less than the minimal required R_{AB}^{opt}, so that this transition could not happen for lack of resources:

if $\mathcal{E}_{AB}^{real} < \mathcal{E}_{AB}^{\max} \;\Rightarrow\; R_{AB}^{real} < R_{AB}^{opt}$; however, this relation can only be: $R_{AB}^{real} \geq R_{AB}^{opt}$.

These considerations show that the only feasible way for nature to attain a macrostate B started in a macrostate A of an ensemble is to minimise the resource R_{AB} required for this transition and to maximise the entropy $\mathcal{E}_{AB}$ of ensemble of the related microstates. This confirms the conclusion in sec. 2.1.5 that the principles of maximal entropy and of minimal resources consumption are equivalent.

2.3.3 Universal Method for Description of a Nature Phenomenon?

The considerations in the previous section allow me to suppose that we can apply a universal method for a dynamic description of a nature phenomenon, namely:

1) Choose the terms of nature phenomenon being complementary to each other. These complementary terms shall be observable and measurable; they also may depend on other measurable parameters of this phenomenon.
2) Build a product of these terms and, by this, the related resource of the phenomenon.
3) Apply the principle of least resources consumption (P.2) to this resource.

This method should yield as a result a relation between the parameters of the phenomenon describing the progress of phenomenon state.

2.4 Nascency and Dissolution of Universes

In order to be able to look into this theme, we have first to make an excursus in some philosophical conceptions.

There are material and ideal objects in nature as well as the processes of interaction between them.

What we usually term as ‘matter’, I class among the material objects. We describe them by particles and/or waves and their aggregates. One of the most important properties of material objects is that they exist in time and space.

Among the ideal objects I class, in general, information. Information can take very different forms like, for example, natural laws or psychic and mental forms. One of the most important properties of ideal objects is that they know neither the term ‘time’ nor the term ‘space’: *In these ‘coordinates’ they are ‘eternal’ and ‘infinite’.*

There is an inseparable reciprocal dependency between ideal and material objects of nature:

– Ideal objects (e.g. natural laws) can only be perceived by means of material objects: If material objects did not exist, ideal objects would not have any opportunity to make themselves noticeable.
– Without ideal objects, material objects would absolutely be homogeneous and symmetric, so that they would not react upon any external impact. Consequently, material objects would not be observable, and, thus, in the inexistence.

The interaction process between material and ideal objects has a direct affinity to asymmetry:

– Existence of ideal objects causes asymmetry;
– Existence of material objects, i.e. their being, is observable, if and only if they possess at least one asymmetry: the absolutely symmetric objects cannot react on any impact, because – in order to be able to react on an impact – an object must possess an asymmetry being affine to this impact.

In that way, asymmetry represents the existence form for ideal as well as for material objects. It is in keeping with the conclusion in sec. 2.1.5.

In other words, information provides matter with the *form* of its existence; matter gives information the *content* of its existence.

Now, we can face the actual theme of this section.

As we already determined in sec. 2.3.1, the information value $IV_{j=N}$ of a macrostate of an ensemble decreases in average by 1 bit per time step `j`. At the very end of this evolution, the information value of the last macrostate would be $IV_{j=N_{\max}} = 0$ and nature would not be observable any more, s. sec. 2.2.2.

The philosophical consequence of such a situation would be that ideal objects (e.g. natural laws) would not be perceivable any more, which would contradict their 'eternity'.

Due to this contradiction, I cannot suppose a fundamental end of the entire Creation (of nature). On the contrary, I suppose that the material part of nature (the material Universe) merely changes its form (incl. nascency of other universes) in the course of time, but never disappears completely (i.e. never becomes absolutely symmetric).

We can say that the properties of ideal objects to be 'eternal' and to retain observable/perceivable result in a situation where also the material objects in their entirety never disappear completely. It means there will always be the material objects being not absolutely symmetric and, thus, in the being.

How can a new 'universe' come into being? How can nature avoid that an existing universe does reduce its information value to zero? In such a case, its entropy would take its maximally possible value and, hence, could not grow any more. Due to this, each next microstate of the ensemble would remain equal to the previous, thus making the ensemble deterministic (its indeterminacy would equal zero). Therefore, the universe would become unobservable (s. sec. 2.1.3) and the term 'time' as the distance between two different microstates of nature (s. sec. 1.3) would not be applicable to this universe any more: the universe would pass over into the state of inexistence.

Hypothesis H.2 (wrong-decision as 'universe generator')

One of the plausible opportunities to avoid such a development of nature (to avoid death of nature) is that each next universe originates from a *wrong-decision of an alternative* in the 'current' universe.

What does a wrong-decision of an alternative mean? Each alternative will most probably be decided in such a way that the entropy of the ensemble will be maximised (s. sec. 2.1.5 and 2.3.1). A wrong-decision of an alternative means a deviation from this principle: a wrong-decided alternative is such where another than the most probable decision node has been chosen within the alternative (cf. Figure 1 in sec. 2.2.2); i.e. the decision made for the alternative is rather improbable.

Such a mistake (wrong-decision of an alternative) shall unconditionally have happened earlier than the current universe put itself into inexistence ($IV_{j=N_{\max}} = 0$).

Let $j = N_{err}^{(0)}$ be the number of time steps within the ensemble of the current universe till the first wrong-decision of an alternative (we use here the notation universe$^{(0)}$, $N^{(0)}$, etc. in order to identify the belonging of parameters to the current Universe; the first universe 'born' of the universe$^{(0)}$ together with its parameters we denote as universe$^{(1)}$, parameter$^{(1)}$).

Then, the current universe$^{(0)}$ will until then have analysed $Z(N_{err}^{(0)})$ decision options (s. sec. 2.3.1):

$$Z(N_{err}^{(0)}) \approx \overline{\alpha}^{N_{err}^{(0)}} = e^{N_{err}^{(0)}} .$$

The probability of this wrong-decision of an alternative (one mistake at $Z(N_{err}^{(0)})$ analysed options):

$$p_{err}^{(0)} = Z^{-1}(N_{err}^{(0)}) \approx e^{-N_{err}^{(0)}} . \tag{2.25}$$

Should this wrong-decision of an alternative create a new universe$^{(1)}$, its initial information value (i.e. at its very first time step `l=0`, s. sec. 2.2.2) would be:

$$IV_{l=0}^{(1)} \equiv \mathcal{E}_{\max}^{(1)} - \mathcal{E}_{l=0}^{(1)} = \mathcal{E}_{\max}^{(1)} = -\ln p_{err}^{(0)} \approx -\ln e^{-N_{err}^{(0)}} = N_{err}^{(0)} .$$

Thus,

$$IV_{l=0}^{(1)} = \mathcal{E}_{\max}^{(1)} \approx N_{err}^{(0)} . \tag{2.26}$$

That means that the number of time steps of the current universe till the first wrong-decision of an alternative determines the initial information value of a new universe 'born' by this mistake.

The birth probability of the universe$^{(1)}$ $p_G^{(1)}$ is obviously equal to the probability of the wrong-decision in the universe$^{(0)}$ having entailed the birth of the universe$^{(1)}$:

$$p_G^{(1)} = p_{err}^{(0)} \approx e^{-N_{err}^{(0)}} = e^{-IV_{l=0}^{(1)}} .$$

Thus,

$$\begin{aligned} p_G^{(1)} &\approx e^{-IV_{l=0}^{(1)}}, \\ IV_{l=0}^{(1)} &\approx -\ln p_G^{(1)} = u_G^{(1)}. \end{aligned} \tag{2.27}$$

That means that the probability of birth $p_G^{(1)}$ of a universe and its initial information value $IV_{l=0}^{(1)}$ bijectively correlate: the greater the indeterminacy of birth $u_G^{(1)}$, the greater the initial information value $IV_{l=0}^{(1)}$ of the born universe.

The lifespan of a universe is proportional to its initial information value $IV_{l=0}^{(1)}$, because the universe gains approximately 1 bit entropy at each time step and, accordingly, loses 1 bit of its information value, s. sec. 2.3.1.

To better perceive the dimensions of these processes, let us assume that our current Universe shall approximately exist $3*10^{10}$ years (~ 10^{18} sec.). Then, the Universe will have done approx. $10^{18} \cdot t_p^{-1} = 10^{18} \cdot 1{,}85 \cdot 10^{43} \approx 10^{61}$ time steps until then[25]. It means that its initial information value $IV_{j=0}^{(0)}$, the indeterminacy of its birth $u_G^{(0)}$, its maximally possible entropy $\mathcal{E}_{\max}^{(0)}$ and the resource R^{opt} consumed for its nascency (s. formula (2.21)) amount to approx. 10^{61} bits. That is an immense resource according to our standards. The birth probability for such a voluminous universe is vanishingly small, s. (2.27).

The current universe$^{(0)}$ gives over to the just born universe$^{(1)}$ the information value in the amount of $N_{err}^{(0)}$ ($IV_{l=0}^{(1)} \approx N_{err}^{(0)}$, s. (2.26)).

However, that is exactly the information value having been lost by the universe$^{(0)}$ until then (= the entropy having been generated by the universe$^{(0)}$ until then, s. sec. 2.3.1). Then, the remaining information value can contingently be consumed by the universe$^{(0)}$ until the end.

It means inter alia that the initial information value of the born universe$^{(1)}$ cannot exceed the initial information value of the current universe$^{(0)}$. This leads to the conclusion that if the universe$^{(0)}$ makes the mistake at the very last time step of its being, it passes on to the universe$^{(1)}$ its <u>whole</u> initial information value, so that universe$^{(1)}$ becomes as voluminous as universe$^{(0)}$ was. Should the mistake happen earlier, the new universe$^{(1)}$ becomes 'smaller' than universe$^{(0)}$.

If an alternative is wrong-decided after a few time steps (i.e. $N_{err}^{(0)}$ is small), a very 'small' universe will be born with an accordingly small initial information value $IV_{l=0}^{(1)} \approx N_{err}^{(0)}$. The probability of such a birth is admittedly not as vanishingly small as of the birth of a 'big' universe (s. (2.27)), but the lifespan of such a small universe is vanishingly short: the whole initial information value $IV_{l=0}^{(1)}$ will be exhausted in merely a few time steps. In order that a new universe can exist approx. for 1 sec., it has to be able to perform approx. $t_p^{-1} = 1{,}85 \cdot 10^{43}$ time steps; i.e. it shall accordingly have inherited the initial information value $IV_{l=0}^{(1)} \approx 10^{43}$ bits.

If the universe$^{(0)}$ can continue to exist after the birth of the first universe$^{(1)}$, it can make further mistakes and, hence, 'bear' further universes #2, #3, … . Hereby, it is obviously valid:

$$IV_{l=0}^{(1)} + IV_{k=0}^{(2)} + \ldots + IV_{q=0}^{(q)} \le IV_{j=0}^{(0)}.$$

How do these universes differ from each other?

a) The form of their natural laws will forever remain the same for the reason that they (laws) are inferable from the universal principle of least resources consumption.
b) It may well be that the concrete values of fundamental physical constants of a universe depend on its initial (or perhaps also on its current?) information value $IV_{l=0}^{(1)}$.

[25] t_p is the Planck time, s. sec. 1.3

c) Since the current universe$^{(0)}$ passes over to the just born universe$^{(1)}$ the information value in amount of $N_{err}^{(0)}$, which the universe$^{(0)}$ has lost until then[26], the assumption itself suggests that the just born universe$^{(1)}$ exists 'in parallel' with the current universe$^{(0)}$, but in a different state space.
Since each birth of a universe creates its own microstructure of time and space[27], these 'in parallel' existing universes may have time and space microstructures (s. sec. 1.3 und 3.1) not interfering with each other. Therefore, they cannot mutually perceive each other.

It may represent an interesting program to disprove or to confirm the theses b) and c).

2.5 Experimental (Direct) Verification of the Time-ensemble Postulate

An experimental verification of the time-ensemble postulate P.1 and of the principle of least resources consumption P.2 is obviously possible merely on the small ensembles of microstates (s. Remark 1 in sec. 1.4 and sec. 2.2.2) and, thus, at very small timespans resp. very big energies.

For this, I see the following reason: importance of fluctuations becomes perceivable/measurable first for small ensembles (the order of magnitude at maximal 10^3 microstates). It means that the probability of deviation between *the really instantiated* macrostate (consisting of a few microstates) and *the expected, most probable* macrostate is greater.

Exactly the latter, ***the most probable* macrostate is described by physical theories**.

It means that, for such small ensembles of microstates, it is probable to determine a deviation of a *realised* macrostate from its *theoretically expected* pendant. Such a deviation would confirm the time-ensemble postulate and the postulate of least resources consumption.

The characteristic timespan for such a small ensemble has the order of magnitude maximal $10^3 \cdot t_p$ time steps (= microstates), i.e. $\approx 5 \cdot 10^{-41}$ sec. The equivalent energies lie in the range of at least $10^{-3} \cdot E_p \approx 10^{16}$ GeV (the Planck energy is $E_p \equiv \frac{\hbar}{t_p} \approx 1{,}2 \cdot 10^{19}$ GeV).

At such small timespans resp. big energies, the *realised* macrostate is basically less and less predictable (and, hence, also less and less controllable), because it is fundamentally probabilistic (s. sec. 2.1.3).

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[26] = the entropy having been generated by the universe$^{(0)}$ until then, so that it remains for the universe$^{(0)}$ the information value retained, which the universe$^{(0)}$ can contingently continue to consume until the end (s. this section above)

[27] Each universe manages/maintains its own state space.

2.6 Global Irreversibility of Nature Evolution

Irreversibility of nature evolution is a direct consequence of the complementary characters of the past and the future (s. sec. 2.1.3): since the future is essentially probabilistic and the past essentially deterministic, an inversion of the direction of time progress would entail that the future would proceed over the known, already determined past, which would contradict its probabilistic character.

The following thought also leads to the same conclusion (s. sec. 2.2.2): time progress generates an ensemble of microstates according to the principle of least resources consumption (P.2); i.e. the information value of each next macrostate decreases as rapidly as possible. Therefore, an inversion of the direction of time progress would imply the most rapid growth of the information value of each next macrostate of nature. For this, nature would need evermore information, i.e. evermore different alternatives, whose source does not exist within an already being universe.

One can express this thought even easier:

'Movement' in the future consists in sempiternal decisions of alternatives and each already decided alternative (more precise: the fact that an alternative was decided) is irreversible.

In this context, the real evolution of nature is essentially irreversible.

Then the question arises how this conclusion corresponds with the reversible laws of physics (e.g. the Liouvill equation, the Schrödinger equation, etc.).

The known dynamic laws of physics are reversible in time just because they describe *not the realised*, but merely *the most probable* macrostates of nature; i.e. the macrostates which would be caused by the expected – and mostly happening – decisions of alternatives (s. sec. 2.2.2). On the other hand, the macrostates of the past (being already determined) mostly result from the most probable macrostates when the latter were still in the future. In other words, the laws of physics describe the already decided alternatives constituting the macrostates of the past.

It means that the known dynamic laws describe the past and merely implicitly presume that this behaviour can be extrapolated into the future.

However, such an assumption remains plausible only under certain conditions, for example, if the macrostates of an ensemble consist of statistically many microstates, so that the fluctuations remain insignificant[28].

[28] The relevant considerations in Ilya Prigogine's '*From being to becoming*', chap. 10 are interesting in this context.

3 Space Microstructure

3.1 Space Quanta

Postulate P.3 (the Space-Quanta Postulate):

Space is not a continuum but is discrete. There is an elementary element of space – the space quant (the smallest space interval).

At every time step – i.e. in each microstate of nature –, a material object occupies *exactly one* certain amount of space quanta.

The space-quanta postulate P.3 is understandably closely related to the time-ensemble postulate P.1 from sec. 1.2. It is nothing more than consequent to introduce the space-quanta postulate in this form, if one takes the most profound affinity between space and time into consideration.

A direct consequence of both postulates (P.1 and P.3) is that, **for the time progress by exactly one time step (one time quant), the space being occupied by an object can change by at most one space quant**.

In order to make this consequence clear, let us assume that, for a time progress of exactly one time step, the space being occupied by an object would change by *two* space quanta. For the sake of clarity, let us consider such a small object as occupying merely one space quant.

Since skipping of space quanta is not possible, this assumption would mean that the object should have had an interim state (the initial state A – in the space quant #1, the interim state – in the space quant #2 and the final state B – in the space quant #3). It means that nature would have had a microstate, where the object was in the space quant #2. However, that would contravene the property P.1-2 in sec. 1.3: nature is and stays, in the frame of the elementary time interval, *exactly in the very same* microstate; there are – by definition – no state transitions within a time quant. It means indeed that it is impossible to induce a change of space by more than exactly one space quant at one time step.

Similar to the time quant, the assumption itself suggests that the value of the space quant is the Planck length $l_p \equiv \sqrt{\frac{\hbar G}{c^3}} \approx 1,6 \cdot 10^{-35} m$.

Though space and time have a big affinity to each other, there are also essential differences between them.

First, as discussed in sec. 2.6, time progress knows *only one direction*, namely from the past into the future[29]. That is a direct consequence of the complementary characters of the deterministic past and the probabilistic future.

[29] The microstates of nature run from the future into the past.

Second, as shown in sec. 2.2.2, time cannot pause. Should it happen, the system would be in the absolutely symmetric macrostate, i.e. in the state of inexistence: the system would then be not observable.

Both circumstances do *not* pertain to the space. Therefore, space knows different directions and there is no compulsion for changing the space occupied by an object at *every* time step; i.e. the space occupied by an object can also remain as it is.

Thus, the following three most important properties of space can be inferred from the time-ensemble postulate P.1 and the space-quanta postulate P.3:

(i) there is no compulsion for changing the space occupied by an object at *every* time step; i.e. the space occupied by an object can also remain as it is;

(ii) if a space occupied by an object changes, then at most by exactly one space quant at one time step;

(iii) the space knows several directions, i.e. a change of the space occupied by an object can happen in several directions.

Fundamental Remark 2:

It fundamentally be remarked, that if we bring a geometry into play introducing a certain coordinate system, we should be aware of the circumstance that a coordinate system is an artefact: by introducing a coordinate system, we basically introduce a set of parameters (coordinates and their mutual relations) being measurable for us (e.g. length, duration, angle, charge, etc.). By this measurability, we consider such parameters as observables. For being an artefact, a coordinate system can be chosen more or less deliberately: it is rather the question of our anthropocentric convenience, of our measuring devices; inter alia, a coordinate system can also be continuous.
Neither the time microstructure (P.1) nor the space microstructure (P.3) can be impacted by a coordinate system, by a geometry.

3.2 Space Translations

Now, we can formalise spacial translation with respect to the properties of space as listed above in sec. 3.1. For the sake of clarity, let us consider a small object occupying merely one space quant and moving one-dimensionally[30].

At every time step `j`, one of the following events concerning space translation can occur:

- either there is no movement, i.e. no space translation: the object occupies the same space quant as in the previous microstate of the ensemble,
- or there is a space translation by exactly one space quant either in one or in the reverse direction[31] (one-dimensional case): the object 'jumps' into the next space quant.

After `N` time steps, the object will have performed

[30] 'One-dimensionally' has here *no geometric* sense: it merely means that the current spacial position of an object is representable by *only one* position number (one can imagine that as a chain node).

[31] This is *not a geometric* direction, but a direction on a 'chain': the current position number of an object can either increment or decrement.

$$N_r = \sum_{j=1}^{N} r_j \qquad (3.1)$$

space translations, whereby r_j is defined as follows:

$r_j = 0$, if no space translation occurred at the time step `j`,

$r_j = +1$, if a space translation occurred in a chosen direction (let us label it as 'right' - the position number of the object has increased) at the time step `j` (by one space quant (one space step)),

$r_j = -1$, if a space translation occurred in the reverse direction (let us label it as 'left' - the position number of the object has decreased) at the time step `j` (by one space quant (one space step)).

In general, it means that the vector r_j represents a sequence of zeros and ±ones, e.g. $r_j(1,0,0,0,1,1,-1,0)$. For this example with altogether 8 time steps (`N=8`), the value of space translation N_r amounts to +2 space translations; i.e. the object moved off from its original position (`j=0`) by 2 space quanta to the 'right'. At the time steps `j=2,3,4,8`, space translations were omitted.

For two special cases of space translations it applies that:

a) If $r_j = 0$ for $\forall j = 1 \div N$ for an object, then – from (3.1) – $N_r = 0$ (no space translation).

b) If $r_j = +1$ or $r_j = -1$ for $\forall j = 1 \div N$ for an object, then $N_r = +N$ or $N_r = -N$, respectively. In this case, there is exactly one space translation at each time step, which also represents the maximally possible value: a material object cannot move quicker than exactly one space step per one time step (s. sec. 3.1).

It means that the velocity of space translations (per time step)

$$\upsilon_r(N) \equiv \frac{N_r}{N} = \frac{1}{N}\sum_{j=1}^{N} r_j \qquad (3.2)$$

possesses a *natural* upper limit: if $N_r = \pm N$, then $\upsilon_r(N) = \pm 1$ space translation per time step.

The next question I would like to consider concerns *relative* space translations, i.e. how a space translation is perceived by an observer.

Let us notionally imagine that all space quanta are unambiguously enumerated (#1, #2, #3, …). Then, each possible space translation is discernible, namely by the currently translating object as well as by another, external object. It is determinable that either there has been no space translation (the 'position number' of the current space quant has not changed) or there is a space translation (the 'position number' of the current space quant differs by 1 from the previous one).

It does **not** contradict Einstein's relativity principle: indeed, though each possible space translation is discernible (the 'position number' of the current space quant is always determinable), physical laws do not depend on a concrete value of this ‘position number’; i.e. they remain invariant in this regard.

It shall be noted here that the term ‘acceleration’ is not applicable for this consideration: the process of space translation – i.e. ‘jumping’ of an object from one space quant to the next one – is a discrete process, where there can be no interim states, neither in time nor in space (postulates P.1 and P.3). Therefore, there is no continuous velocity for such discrete ‘jumps’ and, thus, also no acceleration (as its first derivation).

Now, let us define *relative* space translation as follows: if a space translation occurred either for the translating object or for an external observer, a relative space translation has also happened. It means there is always a relative space translation, namely ±1 at one time step, unless neither the translating object nor an external observer experience a space translation at the time step `j` (then the relative space translation =0).

Thus, *relative* space translation merely depicts the *fact*, whether – for a pair object-observer – a space translation occurred either for the translating object or for the external observer at the time step `j`.

This definition of *relative* space translation can be depicted by the following addition rule for r_j:

Space translation $r_j^{(1)}$ of an external observer	Space translation $r_j^{(2)}$ of the translating object	*Relative* space translation $r_j^{(12)}$
0	0	0
0	+1	+1
0	-1	-1
+1	0	+1
+1	+1	+1
+1	-1	+1
-1	0	-1
-1	+1	-1
-1	-1	-1

Let us here consider some examples to illustrate the addition rule ($j = 1 \div 5$).

a) Supposing, $r_j^{(1)}(1,0,0,0,1)$ ($N_r^{(1)} = 2$) and $r_j^{(2)}(1,0,1,1,0)$ ($N_r^{(2)} = 3$). Then $r_j^{(12)}(1,0,1,1,1)$ ($N_r^{(12)} = 4$).

b) For $r_j^{(1)}(0,0,0,0,0)$ ($N_r^{(1)} = 0$) and $r_j^{(2)}(1,1,1,1,1)$ ($N_r^{(2)} = 5$) we gain $r_j^{(12)}(1,1,1,1,1)$ ($N_r^{(12)} = 5$).

c) For $r_j^{(1)}(0,0,0,0,1)$ ($N_r^{(1)} = 1$) and $r_j^{(2)}(1,1,0,0,0)$ ($N_r^{(2)} = 2$) we gain $r_j^{(12)}(1,1,0,0,1)$ ($N_r^{(12)} = 3$).

This definition of relative space translation entails that the latter is not a linearly-additive quantity. If, for example, an object experiences a space translation at each time step (s. example b) above), then a relative space translation always happens, namely irrespective of space translations of an observer. In case the object as well as the observer experience a space translation very 'seldomly' (i.e. $r_j^{(1)}$ and $r_j^{(2)}$ have mainly zeros as component values and are very long vectors ($N \gg 1$)[32]), the probability of seldom space translations (ones) happening at *different* time steps `j` (i.e. not simultaneously) is very high. In such a constellation, the relative space translation is practically linearly-additive, because the space translations (ones) of the object and of the observer do not temporally overlap each other (s. example c) above).

[32] Please do not forget that 'very long' vectors still matter at, for example, N>1000 time quanta merely > 10^{-41} sec.

4 Physical Theories as Consequences of the Principle of Least Resources Consumption

This chapter aims to show that several fundamental relations of physics can be derived from the time-ensemble postulate (P.1), from the postulate of least resources consumption (P.2), from the space-quanta postulate (P.3) and from assumptions about a concrete geometry of spacetime.

As already discussed in sec. 2.1.5, Hamilton's principle of least action can directly be deduced from the postulate of least resources consumption, so that we can continue further considerations directly with Hamilton's principle.

This chapter does not aim to *mathematically exactly* deduce fundamental equations of physics from Hamilton's principle – that can be found in various books. Rather, this chapter shall *qualitatively* demonstrate how to *physically* apply Hamilton's principle, which implicit or explicit assumptions – inter alia also of spacetime properties – one has to make and which constrains for the resulting depiction of nature these assumptions connote, as well as which physical sense various physical limits have.

If a system transits from a macrostate A [the time step `j=1`] into another macrostate B [the time step `j=N`], the action[33] needed for that can be calculated as[34]

$$S_{AB} = t_p \sum_{j=1}^{N} \mathcal{L}(\mathrm{q}, \dot{\mathrm{q}}), \tag{4.1}$$

whereby $\mathcal{L}(\mathrm{q}, \dot{\mathrm{q}})$ is the related Lagrangian that is summed up over all relevant microstates of the ensemble (`j = 1` to `N` time steps); $(\mathrm{q}, \dot{\mathrm{q}})$ are generalised 'coordinates' and corresponding generalised 'velocities'.

Or, if one divides this equation by `N` and considers that $N \cdot t_p = t_{AB}$:

$$\frac{S_{AB}}{t_{AB}} = \frac{1}{N} \sum_{j=1}^{N} \mathcal{L}(\mathrm{q}, \dot{\mathrm{q}}). \tag{4.2}$$

Due to the principle of least action, dynamic equation for this change of state must fulfil the following condition:

$$\delta S_{AB} = 0, \tag{4.3}$$

whereby δ signifies a small variation and the states A and B are fixed.

If one introduces a *continuous* coordinate system, the sum can be replaced by an integral:

[33] German: Wirkung

[34] provided that the Planck time t_p is independent of each single time step.

$$S_{AB} = \int_{t_A}^{t_B} \mathcal{L}(\mathrm{q}, \dot{\mathrm{q}}) dt \,. \tag{4.4}$$

Then the condition (4.3) gets the form

$$\delta S_{AB} = \delta \int_{t_A}^{t_B} \mathcal{L}(\mathrm{q}, \dot{\mathrm{q}}) dt = 0 \,. \tag{4.5}$$

The well-known Lagrange equation describing dynamic evolution of the state of an object results from (4.4) and (4.3) (subject to differentiability!):

$$\frac{d}{dt} \frac{\partial \mathcal{L}(\mathrm{q}, \dot{\mathrm{q}})}{\partial \dot{\mathrm{q}}} = \frac{\partial \mathcal{L}(\mathrm{q}, \dot{\mathrm{q}})}{\partial \mathrm{q}} \,. \tag{4.6}$$

It should be noted here that *differentiability* represents a *very strongly constraining property* in this context: it means here an assumption about a *continuous* spacetime at least in vicinity of differentiating, which *contradicts* the generally discrete character of spacetime (postulates P.1 and P.3). This assumption also entails *reversibility* of time.

4.1 Classical Dynamics

Classical mechanics considers object states in principle as continuous, i.e. one always finds at least one additional object state between two given states of an object.

Also, the time microstructure (cf. sec. 1.2) and, thus, the time-ensemble postulate do not play any role for classical mechanics, because the classical consideration of processes presumes a limit, namely $\hbar \to 0$. It means, inter alia, that this consideration is valid for the timespans $\gg$ as Planck time $t_p \equiv \sqrt{\frac{\hbar G}{c^5}} \approx 5,4 \cdot 10^{-44} s$, so that discrete time progress is not perceived at all by classical consideration: time flow is there described as *continuous*. Since such a defined time coordinate ignores the time microstructure, the solution of alternatives (sec. 2.2.2) and, thus, being probabilistic of the future also get lost: the time coordinate becomes reversible.

Likewise, the discrete space microstructure (cf. sec. 3.1) does not play any role for classical consideration, because this microstructure would not exist at the limit $\hbar \to 0$ and space would be a *continuum*.

One assumes Euclidean geometry (i.e. a continuous, not curved coordinate system ($\boldsymbol{r}$, t)), see the *Fundamental Remark 2* in sec. 3.1.

Interestingly, as one can learn from (2.6) in sec. 2.1.3 and (2.23) in sec. 2.3.2, the classical limit $\hbar \to 0$ would imply an infinitely large resources consumption of nature, if the latter had actually realised the constellation $\hbar = 0$. A continuous spacetime would – upon a complete consideration, s. discussion in sec. 4.3 – enforce the limit $c \to \infty$.

For classical dynamics, the Lagrangian is $\mathcal{L}(\mathrm{q}, \dot{\mathrm{q}}) = T - V$ (T is kinetic and V – potential energy). Hence, we can vary action S_{AB} as follows:

$$\delta S_{AB} = \int_{t_A}^{t_B} [-m\ddot{q}_{opt} - \nabla V(q_{opt})] \delta q(t) dt \,,$$

whereby q_{opt} represents the sought optimal pathway of trajectory from A to B and $\delta q(t)$ – a small variation of this trajectory about this optimal pathway q_{opt}. Since $\delta S_{AB} = 0$ must be independent of this variation $\delta q(t)$, it means that

$$[-m\ddot{q}_{opt} - \nabla V(q_{opt})] = 0 \tag{4.7}$$

or

$$m\ddot{q}_{opt} = -\nabla V(q_{opt}) .$$

This dynamic equation is nothing else than $\mathbf{F} = m\mathbf{a}$ – Newton's second law.

Thus, we come to the conclusion that

(i) classical dynamics can be derived from the principle of least resources consumption (P.2) and from the implicit assumption about continuous object states and continuous spacetime;

(ii) predictions of classical dynamics remain valid as long as object states and spacetime can be considered as continuous, i.e. inter alia if $mcr \gg \hbar$ (continuity of states) and $t \gg t_p$ and $r \gg l_p$ (continuity of spacetime).

4.2 Quantum Mechanics

Time microstructure is defined by the time-ensemble postulate in sec. 1.2 and space microstructure by the space-quanta postulate in sec. 3.1, whereby no assumptions about a concrete geometry were made there at all.

The phase-translation equation $i\frac{\Delta C_j(\Phi_j)}{\Delta \Phi_j} = -\sum_k \sigma_{jk}(\Phi_j) C_k(\Phi_k)$ (s. (6.5) in annex A.2, sec. 6.2) is also 'geometry-neutral'.

Now, we introduce a time coordinate *t* so that $t \equiv N \cdot t_p$, whereby *N* represents time steps and t_p – the Planck time. This coordinate shall possess one more property: differentiating with respect to it shall be possible. This condition is fulfilled, for example, by a *continuous* time coordinate. Of course, this property directly contradicts the definition $t \equiv N \cdot t_p$, which merely means – from the physical point of view – that the introduction of such a time coordinate is an approximation – namely neglecting the time microstructure. Differentiability with respect to *t* is also responsible for time reversibility in the resulting representations.

This is a fundamental contradiction of 'classical' quantum mechanics: it considers *discrete* object states (due to $\hbar \neq 0$)[35] in *continuous* spacetime. Since such a defined time coordinate ignores time microstructure, decision of alternatives (sec. 2.2.2) and, thus, probabilistic character of the future also get lost: the time coordinate becomes reversible.

[35] $\hbar$ reflects the discreteness of object states.

This approximation is valid, if consideration is for the timespans being greater than the Planck time $t_p \approx 5,4 \cdot 10^{-44} s$ by at least circa one order of magnitude (which already represents a much better degree of fineness than the classical limit $\hbar \to 0$). It means, inter alia, that 'classical' quantum mechanics might render predictions deviating from reality, if timespans are sufficiently short (shorter than circa $100 \cdot t_p$).

One assumes Euclidean geometry (i.e. a continuous, not curved coordinate system (***r***, *t*)), see the *Fundamental Remark 2* in sec. 3.1.

Dynamic equations in the framework of quantum mechanics can be derived from the principle of least action (and, ultimately, from the postulate of least resources consumption), cf. chap. 4, formula (4.5): $\delta S_{AB} = \delta \int_{t_A}^{t_B} \mathcal{L}(\mathrm{q}, \dot{\mathrm{q}}) dt = 0$. For the first time, this was conducted by Richard Feynman, cf. [7].

For stationary states (the states are changing in time, against what the observables retain with respect to time: $\frac{dC_j}{dt} = 0$) of a system evolving according to Hamilton's principle, $-\frac{\partial S}{\partial t} = H$ is a valid equation. Substituting it into the phase-translation equation (s. (6.5) in annex A.2) yields the well-known quantum mechanical dynamic equation for temporal evolution (Schrödinger's equation in time representation):

$$i\hbar \frac{\partial C_j(t)}{\partial t} = \sum_k H_{jk}(t) C_k(t), \tag{4.8}$$

whereby $H_{jk}(t)$ is Hamilton's matrix; $C_j(t) \equiv \langle j | \psi(t) \rangle$ is the amplitude of being the system in the basis microstate j at the time *t*.

Note that the equation (4.8) is affine to the Hamilton-Jacobi differential equation in classical mechanics. It is easily understandable due to the fact that the Hamilton-Jacobi differential equation is also derived from the principle of least action. If one continues this analogy, one comes to the conclusion that action S_j is commensurate with the term $-i\hbar\psi_j$. On the other hand, $\frac{S_j}{\hbar} \sim iu_j(\mathrm{ampl})$ (s. formula (2.6) in sec. 2.1.3). From this it follows that the wave function of a state ψ is affine to indeterminacy u of this state.

Schrödinger's equation in position representation

Now, we also introduce a space coordinate *r* so that $r \equiv N_r \cdot l_p$, whereby N_r represents space translations and l_p is the Planck length. This coordinate shall also allow differentiating with respect to it. This condition is fulfilled by, for example, a *continuous* space coordinate. Exactly as in the case of a time coordinate, it implies neglecting the space microstructure. This approximation is valid, if consideration is for the distances being greater than the Planck length $l_p \equiv \sqrt{\frac{\hbar G}{c^3}} \approx 1,6 \cdot 10^{-35} m$ by at least circa one order of magnitude (which already represents a much better degree of fineness than the classical limit $\hbar \to 0$). It means, inter alia, that 'classical' quantum mechanics might render predictions deviating from reality, if distances are sufficiently short (shorter than circa $100 \cdot l_p$).

It is well-known that Schrödinger's equation in position representation is inferable from the quantum mechanical dynamic equation (4.8)[36]. Formally, it is made by substitution of $\hat{H} = -\frac{\hbar^2}{2m}\Delta + V(\mathbf{r})$ in (4.8).

But it is interesting that Schrödinger's equation in position representation can also be deduced from *qualitative* physical considerations ([6], Chapter 16 "The dependence of amplitudes on position").

As already discussed in sec. 2.2.2, nature creates an ensemble of microstates while time progresses (by decision of alternatives). From the principle of least action, it follows that single microstates of the ensemble are almost equiprobable. This is a consequence of the circumstance that each alternative is most probably decided in a way that maximises the entropy of the ensemble (s. sec. 2.1.5 and 2.3.1), i.e. the decision made for the alternative is very probable.

Since these microstates of the ensemble are the smallest 'entities' of nature (there are no 'interim states', cf. P.1 and P.3), they represent *natural basis states* (in the quantum mechanical sense) of nature.

For the sake of clearness of further description, let us assume that spacial movement is only possible along a 'chain' – depictured by coordinate x: a spacial microstate of the ensemble 'diffuses' from one 'node' of chain to a neighbour 'node'.

Let $C_j(x_j) \equiv \langle j \mid \psi \rangle$ be the amplitude of being the system in the basis microstate j (i.e. at the time step j) in the node x_j of chain.

Since transitions from one microstate of the ensemble to another one are almost equiprobable and no spacial 'node' can be skipped[37], the amplitude $C_j(x_j)$ practically depends on merely the next 'neighbours' of the spacial chain – on the left one and on the right one –, whereby there is no preferred direction.

Now, we can apply the equation (4.8) taking into account all these assumptions (instead of variable *t* at the time step j, we here use the respective position x_j):

$$i\hbar \frac{\partial C_j(x_j)}{\partial t} = E_{jj} C_j(x_j) - E_{(j-1)j} C_{j-1}(x_{j-1}) - E_{(j+1)j} C_{j+1}(x_{j+1}) . \tag{4.9}$$

Here, $E_{jj} \equiv E_0$ is an abstract energy value of some sort, if there is no spacial change of microstate in the node x_j (no space translations); $E_{(j-1)j}$ and $E_{(j+1)j}$ are the energy values associated with space translations from node x_j to the respective nodes x_{j-1} and x_{j+1}. Since

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[36] s. e.g. [7], chap. 4; [12], chap. 3, §17

[37] cf. sec. 3.1: '…for the time progress by exactly one time step (one time quant), the space being occupied by an object can change by at most one space quant'.

there is no preferred direction – left or right –, we can write $E_{(j-1)j} = E_{(j+1)j} \equiv E_1$ (one space translation).

Now, we transform (4.9) as follows:

$$i\hbar\frac{\partial C_j(x_j)}{\partial t} = (E_0 - 2E_1)C_j(x_j) + E_1[2C_j(x_j) - C_{j-1}(x_{j-1}) - C_{j+1}(x_{j+1})]. \qquad (4.10)$$

If one expands $C_{j\pm1}(x_{j\pm1})$ into the Taylor series and leaves the terms till inclusively $(\Delta x)^2$, one gains (we use here the property of differentiability with respect to space coordinate):

$$C_{j\pm1}(x_{j\pm1}) \approx C_j(x_j) \pm \left.\frac{dC}{dx}\right|_{x=x_j} \cdot \left|x_{j\pm1} - x_j\right| + \frac{1}{2}\left.\frac{d^2C}{dx^2}\right|_{x=x_j} \cdot (x_{j\pm1} - x_j)^2. \qquad (4.11)$$

Since E_0 represents merely an abstract energy value, we can choose this calibration in such a way that $E_0 - 2E_1 \equiv 0$. Substituting this calibration and the expansion (4.11) into equation (4.10), we gain

$$i\hbar\frac{\partial C_j(x_j)}{\partial t} \approx -E_1 \cdot \left.\frac{d^2C}{dx^2}\right|_{x=x_j} \cdot (x_{j\pm1} - x_j)^2 = -E_1 \cdot \left.\frac{d^2C}{dx^2}(\Delta x)^2\right|_{x=x_j}. \qquad (4.12)$$

The value E_1 correlates with probability of space translations from x_j to x_{j-1} and to x_{j+1}: if $E_1 = 0$, there would be no space translations. If $(\Delta x)^2 \to \infty$, there would also be no space translations.

The distance (space interval) between the neighbouring nodes x_j and $x_{j\pm1}$ is the Planck length l_p. The energy of a single space translation might be about half of the Planck energy $E_1 \approx \frac{1}{2}E_p \equiv \frac{1}{2}m_p c^2$. Hence, $E_1 \cdot (\Delta x)^2 \approx \frac{1}{2}E_p l_p^2 = \frac{\hbar^2}{2m_p}$, whereby $m_p \equiv \sqrt{\frac{\hbar c}{G}} \approx 2{,}2 \cdot 10^{-8} kg$ is the Planck mass.

Eventually, we get from (4.12)

$$i\hbar\frac{\partial C_j(x_j)}{\partial t} \approx -\frac{\hbar^2}{2m_p} \cdot \left.\frac{d^2C(x)}{dx^2}\right|_{x=x_j}. \qquad (4.13)$$

The well-known Schrödinger's equation in position representation looks as follows:

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \cdot \Delta\psi + V \cdot \psi. \qquad (4.14)$$

That (4.13) and (4.14) are closely related to each other is obvious.

Thus, we come to the conclusion that

(i) Schrödinger's equation in time representation (4.8) can be derived from discrete microstates of the ensemble in time and from the principle of least resources consumption (P.2);

(ii) discrete microstates of the ensemble in space are additionally to be taken into account for the derivation of Schrödinger's equation in position representation (4.14);

(iii) predictions of 'classical' quantum mechanics might deviate from reality, if timespans and/or distances are sufficiently short (shorter than circa $100 \cdot t_p$ and $100 \cdot l_p$, resp.). This is based on a contradictive consideration of *discrete* microstates of the ensemble and of *continuous* spacetime.
The mathematical methods of discrete Markov chains may be well appropriate also to depict the discrete character of spacetime. Probably, quantum mechanical consideration can be extended in such a way.

4.3 Theories of Relativity

Time microstructure is defined by the time-ensemble postulate in sec. 1.2, and space microstructure by the space-quanta postulate in sec. 3.1, whereby no assumptions about a concrete geometry were made there at all (s. also the remark concerning coordinate systems as artefacts in sec. 3.1).

Now, we introduce the following coordinate system *(r, t)*:

$$r \equiv N_r \cdot l_p \,,\; t \equiv N \cdot t_p \,,$$

whereby l_p and t_p are the Planck length and Planck time, resp., N and N_r represent time steps and space translations, resp., cf. sec. 3.2.

Let us now define translation velocity v_r in these coordinates as

$$\mathrm{v}_r \equiv \frac{\Delta r}{\Delta t} = \frac{l_p}{t_p}\frac{\Delta N_r}{\Delta N} = c\frac{N_r}{N} = c \cdot \upsilon_r(N) \,, \qquad (4.15)$$

whereby (3.2) was regarded here.

In sec. 3.2 we have already determined that $\upsilon_r(N)$ has a natural limit, namely $\upsilon_r(N) = \pm 1$: that corresponds to the situation, where exactly one space translation occurs at every time step. It follows from this and from (4.15) that v_r also has a natural limit, namely $\mathrm{v}_r = c$ (absolute value).

It means that the translation velocity v_r has the natural maximal value $\mathrm{v}_r = c$, **which can be exceeded by no means due to the fact that it cannot occur more than exactly one space translation at each time step**[38]**. A material object can move not quicker than** $\frac{l_p}{t_p} = c$[39].

[38] A consequence of the postulates P.1 and P.3, s. sec. 3.1.

[39] If one introduces a 'physically natural' system of units, where the quantities of time- and space-quanta serve as etalons (i.e. $t_p = 1$ and $l_p = 1$), the maximal possible translation velocity is $c = 1$ space translation per a time step.

Here, Einstein's principle of existence of a maximal velocity for propagation of interactions represents a consequence of the time-ensemble postulate and space-quanta postulate.

What does the classical limit $c \to \infty$ mean in this context? From the physical point of view, it would mean that not maximally exactly one space translation, but arbitrarily many space translations at each time step would be possible. It would in turn imply that any number of interim states between two neighbouring microstates of the ensemble would be possible. It would mean that time quant and/or space quant could be arbitrarily small, which would correspond to a time- or space-continuum, resp. Besides this, nature would also consume arbitrarily many resources for creating arbitrarily many interim microstates (approx. 1 bit for each microstate, cf. sec. 2.3.1).

It means that the classical limit $c \to \infty$ presupposes not a *discrete*, but a *continuous* spacetime. Besides this, it would mean an infinite consumption of the resources of nature, if the latter had actually realised the constellation $c \to \infty$. In this regard, the limit $c \to \infty$ is similar to the limit $\hbar \to 0$ (cf. sec. 4.1).

This is a fundamental contradiction within the theories of relativity (in the Special as well as in the General): they correctly constrain the maximal velocity of space translations ($c < \infty$)[40] and simultaneously use continuous spacetime, which implies the classical limit $c \to \infty$. Since such a defined time coordinate ignores the time microstructure, decision of alternatives (sec. 2.2.2) and, thus, being probabilistic of the future also gets lost: the time coordinate becomes reversible.

Just as for quantum mechanics, this approximation is valid, if consideration is for the timespans or distances being greater than the Planck time t_p resp. Planck length l_p by at least circa one order of magnitude. It means, inter alia, that the theories of relativity might render predictions deviating from reality, if timespans and/or distances are sufficiently short (shorter than circa $100 \cdot t_p$ resp. $100 \cdot l_p$).

From (3.2) with the definition of coordinates (*r, t*) above, we obtain

$$N_r \equiv \upsilon_r(N) \cdot N \;\Rightarrow\; \frac{r}{l_p} \equiv \upsilon_r(N) \frac{t}{t_p} \;\Rightarrow\; r \equiv \upsilon_r(N) \cdot c \cdot t\,,$$

thus,

$$r \equiv \upsilon_r(N) \cdot c \cdot t \tag{4.16}$$

or

$$r^2 - \upsilon_r^2(N) \cdot c^2 t^2 \equiv 0\,. \tag{4.17}$$

Generally, r can be represented by using further coordinates x_μ as follows:

$$r^2 \equiv \sum_{\mu\nu} g_{\mu\nu} x^\mu x^\nu\,,$$

[40] admittedly, by a special postulate without stating any theoretic-physical rationale for this

whereby $g_{\mu\nu}$ is the metric tensor.

4.3.1 Special Theory of Relativity

One assumes Euclidean geometry (i.e. a continuous, not curved coordinate system ($\boldsymbol{r}$, t)), see the *Fundamental Remark 2* in sec. 3.1.

For the planar, Euclidean geometry (three-dimensional)

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

so $r^2 = x_1^2 + x_2^2 + x_3^2$. Using the notation $x_1 \equiv x,\ x_2 \equiv y,\ x_3 \equiv z$ and from (4.17) we obtain

$$x^2 + y^2 + z^2 - \upsilon_r^2(N) \cdot c^2 t^2 \equiv 0\,. \tag{4.18}$$

This identity shall be valid irrespective of the coordinate system chosen (which merely represents an artefact) and for all the allowed values of $\upsilon_r(N)$, i.e. also for $\upsilon_r(N) = 1$. In this way, we gain the well-known initial relation for the Minkowski interval:

$$x^2 + y^2 + z^2 - c^2 t^2 \equiv 0\,, \tag{4.19}$$

which shall remain invariant in all coordinate systems.

All transformation formulas for coordinates as well as Einstein's relation for addition of velocities can be derived thereof using the postulate of special relativity[41] (see e.g. [2]).

Dynamic equations in the frame of the special theory of relativity can be derived from the principle of least action (and, ultimately, from the postulate of least resources consumption) (cf. sec. 4, formula (4.5): $\delta S_{AB} = \delta \int_{t_A}^{t_B} \mathcal{L}(\mathrm{q}, \dot{\mathrm{q}}) dt = 0$).

The related Lagrangian for the relativistic case is:

$$\mathcal{L}(\mathrm{q}, \dot{\mathrm{q}}) = -m_0 c^2 \sqrt{1 - \frac{\dot{\mathrm{q}}^2}{c^2}} - V_{extern}(\mathrm{q}, \dot{\mathrm{q}})\,, \tag{4.20}$$

whereby $V_{extern}(\mathrm{q}, \dot{\mathrm{q}})$ is the potential of an external field (s. e.g. [11], chap. 2, §8-9).

4.3.2 General Theory of Relativity

Hilbert, Lorentz and Einstein were already able to derive the general theory of relativity from Hamilton's principle of least action (which, in turn, is derivable from the postulate of least resources consumption, s. sec. 2.1.5, postulate P.2)[42]:

[41] Physical laws must retain their form in all inertial systems.

$$\delta S_{AB} = \delta \int_{t_A}^{t_B} \mathcal{L}(\mathrm{q}, \dot{\mathrm{q}}) dt = 0 ,$$

see sec. 4, formula (4.5); $\mathcal{L}(\mathrm{q}, \dot{\mathrm{q}})$ is the Lagrangian.

Einstein postulated this in the form that the movement in 4-D space (i.e. in spacetime) happens on geodetic lines. Geodetic lines represent the shortest paths from a point A to another point B and are formally defined as follows:

$$\delta \int_{A}^{B} ds = 0 ,$$

whereby the invariant of the interval ds is defined in the form $ds^2 \equiv \sum_{\mu\nu} g_{\mu\nu} x^\mu x^\nu$ ($g_{\mu\nu}$ is the metric tensor).

Mind you, a movement on a geodetic line is *equivalent* to a trajectory which can be calculated from Hamilton's principle, whereby the Lagrangian formally looks as follows (see [16], § 15):

$$\mathcal{L}(\mathrm{x}, \dot{\mathrm{x}}) = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu .$$

It means that the general theory of relativity is also derivable from the principle of least resources consumption.

A few more words on Einstein's postulate of general relativity[43]. The postulate of general relativity is always fulfilled if Hamilton's principle is used as the basis[44]. Then it also means that application of the principle of least resources consumption always allows fulfilling the postulate of general relativity. **Conceivably, one may interpret this interrelation in a way that the postulate of general relativity is one of the *consequences* of the principle of least resources consumption**.

In my opinion, the postulate of general relativity is a direct consequence of the circumstance that a geometry – being actually defined by a corresponding coordinate system – represents an artefact and can arbitrarily be chosen (s. sec. 3.1). In contrast, the spacetime microstructure is defined by the postulates P.1 (the time-ensemble postulate) and P.3 (the space-quanta postulate), s. sec. 1.2 resp. 3.1, whereby no assumptions about a concrete geometry were made there. Since a geometry represents an artefact and thus an instrument of description, physical laws must not depend on a concrete geometry. From this consideration we can infer that a gravitational field does not impact the *geometry* of spacetime, but, instead, the *actual microstructure* of spacetime. This is also indicated by the fact that the gravitational constant *G* is connected with the values of time quant as well as of space quant and, thus, also determines spacetime discreteness[45].

[42] see [4], [15], [9] and [16]

[43] physical laws must retain their form in all possible Gauss' coordinate systems, which can be defined in the spacetime, i.e. they must show general coordinate-covariance.

[44] see [3]

[45] the quotient G/c reflects, from my point of view, the discreteness of spacetime.

Speculative Considerations

As already discussed in sec. 2.1.4, nature needs to spend at least one action quant in order to create an observable macrostate; i.e. at least one action quant 'falls due' at each time step, whereby this action quant is the minimal action $\hbar$. This circumstance can be recorded as follows: $\min S_{one\_time-step} \equiv \sum_{j=1}^{1} \mathcal{L} \cdot t_p \equiv \mathcal{L} \cdot t_p = \hbar$ (integrating was substituted by summing up – here over a single time step – because time steps do not represent a continuum). From this, the minimal value of the Lagrangian is:

$$\min \mathcal{L} = \frac{\hbar}{t_p} = E_p = m_p c^2 .$$

Since the Lagrangian – as a sort of translation operator – is proportional to the Planck mass m_p, I speculate that the Planck mass represents a measure of the 'inertia' of nature at transitions between its states. Indeed, from the definition for m_p we can derive the following representation:

$$\frac{\hbar}{\frac{G}{c}} = m_p^2 .$$

If one interprets $\hbar$ as the discreteness of *object states* and G/c – as the discreteness of *spacetime*, then the Planck mass m_p is a measure of the 'balance' between the discreteness of object states and the discreteness of spacetime. If object states are almost continuous ($\hbar \to 0$), nature can realise almost unlimitedly many states 'densely' next to each other, so that its 'inertia' is small. If spacetime is almost continuous ($G/c \to 0$), then nature has to perform almost unlimitedly many translations in spacetime in order to reach a certain macrostate, so that its 'inertia' is big.

There is also a representation for the Planck length l_p in the form of the product of the discretenesses:

$$\hbar \cdot \frac{G}{c} = c^2 \cdot l_p^2 .$$

Now, I continue with my speculations and consider it possible that the measure of the 'inertia' of nature for *space translations* represents the *inertial* mass, whereby for *time translations* (time steps) – the *gravitational* mass. This would explain their equivalence being assumed in the general theory of relativity.

One could continue the speculations and assume that the values of the time- and space-quanta - t_p and l_p, resp. - are not constant, but can vary (this presupposes, of course, varying at least one natural constant, for example G, because it reflects the discreteness of spacetime). In such a case, we would perceive our perceivable/observable spacetime[46] as uneven, curved.

If one fixes the time- and space-quanta $t_p = 1$ and $l_p = 1$ as etalons in a 'natural' system of units, one gets to the notion that both the discretenesses $\hbar$ and G/c can vary. We would then perceive the varying of the spacetime discreteness as an uneven, curved spacetime; for how

[46] which merely *reflects* the microstructure of time and space, s. the *Fundamental Remark 2* in sec. 3.1

we would perceive the varying of the discreteness of object states, I do not currently have any notion.

The end of speculative considerations

Thus, we conclude that

(i) Einstein's principle of existence of a maximal velocity for propagation of interactions represents a consequence of the time-ensemble postulate and the space-quanta postulate;

(ii) All transformation formulas for coordinates as well as Einstein's relation for addition of velocities can be derived from the discrete microstates of the ensemble in time (P.1) and in space (P.3) (special theory of relativity);

(iii) Dynamic laws of both the theories of relativity can be derived from the principle of least resources consumption (P.2);

(iv) Predictions of the theories of relativity (special as well as general) might deviate from reality, if timespans and/or distances are sufficiently short (shorter than circa $100 \cdot t_p$ and $100 \cdot l_p$, resp.). It grounds in a contradictive consideration of a *limited* maximal velocity of space translations and of *continuous* spacetime.

4.4 Electrodynamics

The geometry assumed for electrodynamics is identical to the one for special theory of relativity: Euclidean geometry, i.e. a continuous and not curved coordinate system (***r***, *t*), s. sec. 4.3.1. It means that all the properties and constraints resulting from this geometry (see sec. 4.3 for details) are also valid for electrodynamics, inter alia the contradiction between limiting the maximal velocity of space translations ($c < \infty$) and the simultaneous usage of continuous spacetime, which denotes the classical limit $c \to \infty$.

When an electrically charged body with the charge q is moving in an electromagnetic field with the magnetic flux Φ^m from a point A to a point B, the field contribution to the phase change is[47]

$$\Phi_{AB}^{em} = \frac{S_{AB}^{em}}{\hbar} = \frac{q\Phi_{AB}^{m}}{\hbar},$$

whereby S_{AB}^{em} represents the field contribution to the action on the path from A to B.

Magnetic flux is defined as

$$\Phi_{AB}^{m} \equiv \int_{A \to B} \mathbf{A} d\mathbf{s} - \int_{A \to B} \varphi dt \equiv \sum_{i=1}^{4} \mathrm{A}_i \mathrm{x}^i ,$$

[47] Please note here that Φ_{AB}^{em} represents the phase change and Φ_{AB}^{m} – the magnetic flux.

whereby $\mathbf{A}$ is the vector potential and φ – the scalar potential (electrostatic potential); A_i is an equivalent representation in Minkowski space: $A_i \equiv (\varphi, \mathbf{A})$.

Then, the electromagnetic contribution to action (the share of the interaction between the field and the charged particle) is

$$S_{AB}^{em} \equiv q\left[\int\limits_{A\to B} \mathbf{A}d\mathbf{s} - \int\limits_{A\to B} \varphi dt\right] = q\int\limits_{A\to B}\left[\frac{d}{dt}\int\limits_{A\to B} \mathbf{A}d\mathbf{s} - \varphi\right]dt = q\int\limits_{A\to B}\left[\frac{d}{dt}\int\limits_{A\to B} \mathbf{A}\cdot\mathbf{v}dt - \varphi\right]dt = \\ = q\int\limits_{A\to B}\left[\mathbf{A}\mathbf{v} - \varphi\right]dt .$$

The action of the field in itself (without charged particles) is

$$S_{AB}^{ff} \equiv -4\pi\varepsilon_0 c \int\limits_{A\to B} F_{ik}F^{ik} d\Omega ,$$

whereby $F_{ik} \equiv \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}$ is the electromagnetic field tensor and $d\Omega \equiv cdt \cdot dx \cdot dy \cdot dz$ [48].

Thus, the expression for the complete action (also with the share of the free particle) is:

$$S_{AB} \equiv -\int\limits_{A\to B} mcds - \frac{1}{c}\int\limits_{A\to B} A_i j^i d\Omega - 4\pi\varepsilon_0 c \int\limits_{A\to B} F_{ik}F^{ik} d\Omega , \quad (4.21)$$

whereby $j^i = (\rho, \mathbf{j})$ is the current density in Minkowski space.

Now, Hamilton's principle of least action (which is derivable from the postulate P.2) can be applied in two different ways:

a) Either one varies the trajectory of a particle, whereby the field itself remains fixed. In this case, one obtains movement equations for the particle in the field;
b) Or one varies field potentials (which then act as 'coordinates'), whereby the trajectory of the particle remains fixed. In this case, one gains field equations for the field itself.

Thus, if one varies S_{AB} (4.21) with respect to coordinates (usual Lagrange equation), one gains two well-known Maxwell equations (force equations)

$$\vec{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \\ \vec{\nabla} \cdot \mathbf{B} = 0 \quad (4.22)$$

as well as the Lorentz force $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \cdot \mathbf{B})$.

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[48] $(4\pi\varepsilon_0)^{-1}$ reflects, from my point of view, the discreteness of 'charge space'. $(4\pi\varepsilon_0)^{-1}/c$ is the Planck impedance.

If one varies S_{AB} (4.21) with respect to field potentials, one obtains two further Maxwell equations (wave equations)

$$\vec{\nabla}\cdot\mathbf{E} \equiv -\nabla^2\varphi + \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{\rho}{\varepsilon_0},$$
$$\vec{\nabla}\times\mathbf{B} - \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} \equiv -\nabla^2\mathbf{A} + \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = \frac{\mathbf{j}}{\varepsilon_0 c^2} \quad (4.23)$$

as well as the continuity equation for charge- and current densities $\vec{\nabla}\cdot\mathbf{j} = -\frac{\partial\rho}{\partial t}$ (the charge conservation law).

Further details can be found, for example, in [11], chap. 3, §§ 16-17, chap. 4, §§ 26-30 as well as in [5], chap. 15 “The vector potential”, § 5.

Thus, we come to the conclusion that

(i) Electrodynamics can be derived from the principle of least resources consumption (P.2);

(ii) Predictions of electrodynamics (also of quantum electrodynamics) might deviate from reality, if timespans and/or distances are sufficiently short (shorter than circa $100\cdot t_p$ and $100\cdot l_p$, resp.). It grounds in a contradictive consideration of a *limited* maximal velocity of space translations and of *continuous* spacetime.
In this context, I would like to note that classical electrodynamics, which not only neglects the spacetime microstructure, but also considers object states and ‘charge space’ as *continuous* as well as charged particles as geometric points, deviates from reality already at distances much bigger than $100\cdot l_p$, namely at $r \sim (4\pi\varepsilon_0)^{-1}\frac{e^2}{mc^2}$.

4.5 Statistical Physics

The known Poincaré-Misra theorem can be brought to a clear wording: it is impossible to express thermodynamics using deterministic equations of dynamics. It is an exceedingly important statement making understandable, inter alia, the existence of different approaches for describing properties of statistical systems. We will delve on these different approaches below.

a) Statistical approach of incomplete information

This approach is based on deterministic dynamics. The logical chain used there begins with Hamilton’s principle (which is derivable from the postulate P.2). From this, canonical equations are derived, from which, in turn, Liouville’s equation follows; the latter expresses incompressibility of dynamic phase space (q, p). From Liouville's equation, the BBGKY[49] hierarchy can be derived. This hierarchy represents a system of integro-differential equations for partial distribution functions, whereby the distribution function of rank (n) is expressed by the distribution function of rank (n+1). For further details please refer, for example, to [1].

The price for using deterministic dynamics is an *infinite* chain of partial distribution functions in the BBGKY hierarchy. All the constraints that appear by disregarding the discrete spacetime microstructure (s. sec. 4.1 - 4.3) are a further price we need to pay for this approach.

b) Kinetic equations

Kinetic equations for distribution functions generally introduce a collision term being differently modelled from one concrete kinetic equation to another (Boltzmann, Fokker-Planck, Landau), but always assuming a random colliding within a statistical amount of particles. Exactly by using such a collision term, kinetic equations introduce the irreversibility of system evolution. If one neglects the collision term in these equations, one obtains a description of a system in equilibrium: the entropy growth then equals zero. For further information please refer to [13].

Kinetic equations also presuppose a continuous spacetime, so that all the constraints that appear by disregarding the discrete spacetime microstructure need to be taken into account (s. sec. 4.1 - 4.3).

c) Microscopic entropy as operator of thermodynamic evolution

Ilya Prigogine and B. Misra consider dynamics and thermodynamics – based on the Poincaré-Misra theorem – as complementary entities. They introduce operators for microscopic entropy $\hat{M}$ and for system-individual time $\hat{T}$, whereby the operator $\hat{M}$ does not commutate with the dynamic Liouville operator $\hat{L}$.

By applying this approach, the irreversibility of time can be described. A concrete time direction (the selection principle according to Prigogine) is chosen here in a way that the entropy of a system in the future is greater than the entropy of the system in the past. The present, according to Prigogine, does not represent an infinitely small 'point' on the time axis, but a certain timespan (in my opinion, this timespan 'the present' corresponds exactly with the existence duration of a respective process as long as this process remains identical to itself[50]). In the context of this current contribution, the minimally possible duration of 'the present' corresponds to the value of the time quant. For further details, please refer to [17], chap. 8-10.

The Prigogine-Misra approach represents, to my mind, a big step forward in comprehension of thermodynamics itself and its inner relation to deterministic dynamics. Though a discrete spacetime microstructure is not directly addressed there, introduction of the entropy- and time-operators with discrete eigenvalues at least depictures the discrete time microstructure more adequately.

[49] Bogoliubov-Born-Green-Kirkwood-Yvon

[50] in Bohr's sense

d) Gibbs ensembles

Gibbs ensembles shall be considered independently of the previous three essentially different approaches. Gibbs ensembles depend neither on deterministic dynamics nor on kinetics nor on thermodynamics: They can be inferred from the notion of existence of discrete states and from the principle of equal a priori probabilities (s. e.g. [1], chap. 4). This principle represents a *cognitive* component of consideration and predicates how an 'observer' shall handle information, which a system makes about itself available to him.

The principle of equal a priori probabilities is a special case of the principle of maximum entropy[51]. The latter means for statistical physics: "*Amongst all states of a physical system that are compatible with the available knowledge about the system, the state that maximises entropy is to be chosen*", please refer to [10].

The approach having been chosen in the current contribution resolves, inter alia, some essential issues of statistical physics.

First, time irreversibility as a fact by itself directly follows from the complementarity of the probabilistic future and the deterministic past (s. sec. 2.1.3).

Second, the concrete time direction, as we observe it, can also be directly inferred from the complementarity of the probabilistic future and the deterministic past (s. sec. 2.1.3).

The inner 'mechanism' responsible for time irreversibility and the time direction is deciding the alternatives (s. sec. 2.2.2).

The second law of thermodynamics about growth of entropy in the course of time **directly follows from time irreversibility and the concrete time direction** (Clausius, Boltzmann). Therein also lies the distinction to the Prigogine-Misra approach: they *postulate* the already perceivable/observable, known time direction by the selection principle (according to Prigogine) in such a way, that the second law of thermodynamics is fulfilled (s. [17], chap. 10, Entropy Barrier).

Third, the ergodic problem automatically resolves itself by the time-ensemble postulate P.1, s. sec. 1.4.

I would like to conclude this section by noting that observable nature is far away from thermodynamic equilibrium. Indeed, the principle of maximal entropy and of maximal entropy production follows from the postulate of least resources consumption P.2, s. sec. 2.1.5. Stability of an isolated statistical system just corresponds to the maximal entropy production[52]. If one heuristically transfers this statement to nature, one comes to the conclusion that observable nature is far away from equilibrium and simultaneously stable. If one takes the enormous initial information value $IV_{j=0}$ of a historically long[53] existing universe (like, for example, circa 10^{61} bits for our one, s. sec. 2.4) into consideration, this conclusion seems to be a very plausible one.

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[51] A method of Bayesian statistics

[52] cf. [17], chap. 4, sec. *Theory of thermodynamic stability*.

[53] i.e. from the human subjective point of view

5 Summary

Main Postulates

The main conclusions of this treatise are based on an assumption about a non-continuous, discrete spacetime microstructure as well as about a rule that steers transitions between states of nature. This assumption is formulated by way of postulates:

P.1 (time-ensemble):	Time does not progress continuously, but discretely (in time quanta), and each time quantum generates exactly one microstate of nature. This discrete time flow produces an ensemble of microstates.
P.2 (least resources consumption):	The ensembles of microstates (i.e. macrostates) evolve in such a way that the resources of nature required for this are consumed most economically (minimally9.
P.3 (space-quanta):	Space is not a continuum but is discrete. There is an elementary element of space – the space quant (the smallest space interval). At every time step – i.e. in each microstate of nature –, a material object occupies *exactly one* certain amount of space quanta.

Main Conclusions

The most important consequences following from these postulates are:

1) The future and the past possess fundamentally different, complementary characters: the future is *probabilistic*; the past is in contrast *deterministic*.
Observable and existable states of nature can only be probabilistic, but never deterministic.
The Heisenberg uncertainty relations reflect the condition of observability of states and, thus, their property of being probabilistic.
The present might represent a deterministic-probabilistic synthesis because it probabilistically arises and deterministically resigns.

2) An immediate consequence of fundamentally different, complementary characters of the future and the past (probabilistic vs. deterministic) is that time progress is *vectored* and *irreversible*.

3) 'Movement' into the future consists in sempiternal decisions of alternatives[54] and each already decided alternative (more precise: the fact that an alternative was

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[54] There can be binary, trinary, tetrary, and so on alternatives: dimension of an alternative $\alpha \geq 2$;

A deeper reason for the difference between the Shannon and the thermodynamic entropy became comprehendible: the information entropy (Shannon) is defined on the array of exclusively binary ($\alpha = 2$) alternatives; in contrast, the thermodynamic entropy is defined on the array of all alternatives existing in nature ($2 \leq \alpha < \infty$) with their 'natural' distribution $\rho(\alpha)$. This 'natural' distribution causes the average value of dimensions of alternatives $\bar{\alpha} = e$ (Euler's number). *This can be interpreted as the 'physical' sense of Euler's number*.

decided) is irreversible: an act of decision-making conveys and converts the future into the past. In this context, the real evolution of nature is *essentially irreversible*.

4) The only feasible way of nature to attain macrostate B started in macrostate A of an ensemble is such, where the resource R_{AB} required for this transition is minimised and the entropy $\mathcal{E}_{AB}$ of the ensemble of related microstates is maximised (*the principle of most entropy*).

   Hamilton's principle of least action is equivalent to this statement and can be derived from the resources consumption postulate P.2.

   The principle of most entropy means that nature is evolving in such a way that it is producing the most possible entropy. On the other hand, the objects of nature producing maximal entropy are self-organised. Thus, formation of self-organised objects and their associations[55] is rather a very probable way of evolution of nature.

5) The entropy of the nature-ensemble is growing by an average of 1 bit at each time step. Accordingly, the information value $IV_{j=N}$ of the macrostate of the nature-ensemble is decreasing by an average of 1 bit per time step.

6) In the time progress of exactly one time step (one time quant), the space being occupied by an object can change by at most one space quant. It means that the speed of space translations has a *natural* upper limit, namely exactly <u>one</u> space translation per time step.

   Einstein's principle of existence of a maximal velocity for propagation of interactions represents a consequence of the time-ensemble postulate P.1 and the space-quanta postulate P.3.

7) 'Information' as a term is adequately definable as 'alteration of the degree of indeterminacy' in the frame of a system.

8) All currently known physical dynamic equations can be derived from Hamilton's principle and, thus, from the resources-consumption postulate P.2.
   The fact that time in dynamic equations acts as a *reversible* parameter is attributable to the circumstance that the known dynamic laws in fact *describe the past* and merely implicitly *presume* that this behaviour can be extrapolated into the future.

9) Also, thermodynamics and kinetics can be inferred from the resources-consumption postulate P.2 upon additional suppositions concerning a certain direction of time flow and stochastic collisions respectively.
   In the current contribution, time irreversibility and concrete time direction, as we observe it, immediately follow from the complementarity of the probabilistic future and the deterministic past.
   Time irreversibility and concrete time direction lead directly to the second law of thermodynamics about growth of entropy in the course of time.
   The ergodic problem automatically resolves itself by the time-ensemble postulate P.1: in factuality, the averaging over time represents the averaging over the related ensemble being generated by the time progress.

[55] Biological objects also belong to the class of self-organised objects; their associations represent societies.

10) Observable nature is far away from equilibrium and at the same time stable. If one takes the enormous initial information value $IV_{j=0}$ of a historically long[56] existing universe[57] into consideration, this conclusion seems to be a very plausible one.

What Physical Theories Ignore

Classical Dynamics:

The classical limit $\hbar \to 0$ would imply an infinitely big resources consumption by nature, if the latter had realised this constellation $\hbar = 0$ indeed. A continuous spacetime would – upon a complete consideration – enforce the limit $c \to \infty$.

Predictions of classical dynamics remain valid as long as object states and spacetime can be considered as continuous, i.e. inter alia, if $mcr \gg \hbar$ (continuity of states) and $t \gg t_p$ [58] and $r \gg l_p$ [59] (continuity of spacetime).

Quantum Mechanics:

The fundamental contradiction of 'classical' quantum mechanics consists in considering *discrete* object states (due to $\hbar \neq 0$) in *continuous* spacetime. Since such a defined time coordinate ignores the time microstructure, decision of alternatives and, thus, being probabilistic of the future also gets lost: the time coordinate becomes reversible.

It grounds in this contradictive consideration that predictions of 'classical' quantum mechanics might deviate from reality, if timespans and/or distances are sufficiently short (shorter than circa $100 \cdot t_p$ and $100 \cdot l_p$, resp.).

Theories of Relativity and Electrodynamics:

The classical limit $c \to \infty$ presupposes not a discrete, but a continuous spacetime. Besides this, it would mean an infinite consumption of resources of nature, if the latter had actually realised this constellation $c \to \infty$. In this regard, the limit $c \to \infty$ is similar to the limit $\hbar \to 0$.

This is a fundamental contradiction within the theories of relativity (in the Special as well as in the General) and within electrodynamics: they correctly *constrain* the maximal velocity of space translations ($c < \infty$)[60] and simultaneously use *continuous* spacetime, which implies the classical limit $c \to \infty$. Since such a defined time coordinate ignores the time microstructure, decision of alternatives and, thus, being probabilistic of the future also gets lost: the time coordinate becomes reversible.

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[56] i.e. from the human subjective point of view

[57] Like, for example, circa 10^{61} bits for our one

[58] the Planck time

[59] the Planck length

[60] admittedly, by a special postulate without stating any theoretic-physical rationale for this

It grounds in this contradictive consideration that predictions of the theories of relativity and of electrodynamics might deviate from reality, if timespans and/or distances are sufficiently short (shorter than circa $100 \cdot t_p$ and $100 \cdot l_p$, resp.).

Statistical Physics:

The statistical approach of incomplete information (BBGKY-hierarchy) and the approach of kinetic equations neglect the *discrete* microstructure of spacetime and, hence, experience the related contradictions depending on whether a concrete consideration is classical or quantum mechanical.

Merely the approach of microscopic entropy as an operator of thermodynamic evolution (by Prigogine) more adequately depicts the discrete time microstructure and time irreversibility by introduction of entropy- and time-operators.

Only the Gibbs-ensembles approach is independent of assumptions about a concrete spacetime microstructure: it can be inferred from the notion of *discrete* states and from *the principle of equal a priori probabilities*. This principle represents a *cognitive* component of consideration and predicates how an ‘observer’ shall handle information, which a system makes about itself available to him.

What Can Help Us Move Forward on This Path

a) In order to really eliminate the known fundamental contradictions in physical theories, it is not sufficient just to postulate $\hbar > 0$ and/or $\frac{G}{c} > 0$. One should describe *object states and spacetime consequently as discrete entities*: It would necessarily and consistently yield that the discreteness of object states ($\hbar$) and the spacetime discreteness (G/c) are greater than zero. Similarly, it might also be valid for the discreteness of ‘charge space’ $(4\pi\varepsilon_0)^{-1}$.

b) It may be assumed that the approach of discrete Markov chains should be well appropriate to mathematically describe this kind of nature evolution – the discrete progress of time by decisions of alternatives.

c) It would be interesting to research, whether the concrete values of fundamental physical constants of a universe depend on its initial (or perhaps also on its current?) information value $IV_{j=0}$.

d) For a small ensemble of microstates, it is probable to determine a deviation of a *realised* macrostate from its *theoretically expected* pendant. Such a deviation would confirm the postulates of this treatise.
A direct verification of the postulates P.1 – P.3 is obviously only possible on small ensembles of microstates, and, hence, at very short timespans and very big energies respectively. The characteristic timespan for such a small ensemble has the order of magnitude of maximally $10^3 \cdot t_p$ time steps, i.e. $\approx 5 \cdot 10^{-41}$ sec. The equivalent

energies lie in the range of at least $10^{-3} \cdot E_p \approx 10^{16}$ GeV[61].

At such small timespans and big energies respectively, the *realised* macrostate is basically less and less predictable (and, hence, also less and less controllable), because it is fundamentally probabilistic.

[61] E_p – the Planck energy

6 Annex

6.1 A.1: Entropy of a State Generator

Let it be:

– Z – Number of possible choices/states (e.g. a dice with Z faces);

– $I \equiv \log_2 Z$;

– All states are different;

– p_k – probability of the state occurring k , $k = 1,...,Z$, so that $\sum_{k=1}^{Z} p_k = 1$.

– The Shannon entropy of a state generator is defined as $\mathcal{E} \equiv -\sum_{k=1}^{Z} p_k \log_2 p_k$.

Let us define deviation δ_k so, that $p_k = \frac{1}{Z}(1+\delta_k)$. Since $0 \le p_k \le 1$, $\Rightarrow$ $1 \le \delta_k \le Z-1$.

From the condition $\sum_{k=1}^{Z} p_k = 1$ it follows $\frac{1}{Z}(1+\delta_k) = 1$ $\Rightarrow$ $1 + \sum_{1}^{Z} \delta_k = 1$ $\Rightarrow$ $\sum_{1}^{Z} \delta_k = 0$.

Then $\mathcal{E} = -\sum_{k=1}^{Z} \frac{1}{Z}(1+\delta_k) \log_2 \frac{1+\delta_k}{Z}$.

If $\delta_k \equiv 0$ $\Rightarrow$ $\mathcal{E} = \frac{1}{Z} \log_2 Z \cdot \sum_{1}^{Z} 1 = \log_2 Z$, i.e.

$$\boxed{\mathcal{E}(\delta_k \equiv 0) = \log_2 Z \equiv I}.$$

Let us define $\mathcal{E}_k(\delta_k) \equiv -p_k \log_2 p_k = -\frac{1}{Z}(1+\delta_k) \log_2 \frac{(1+\delta_k)}{Z}$.

Then $\mathcal{E}_k(0) = \frac{1}{Z} \log_2 Z$;

$$\mathcal{E}_k'(\delta_k) = \frac{1}{Z} \log_2 \frac{(1+\delta_k)}{Z} - \frac{1}{Z}(1+\delta_k) \frac{Z}{1+\delta_k} \log_2 e \cdot \frac{1}{Z} = -\frac{1}{Z} \log_2 \frac{e(1+\delta_k)}{Z} ; \ \mathcal{E}_k'(0) = \frac{1}{Z} \log_2 \frac{Z}{e} ;$$

$$\mathcal{E}_k''(\delta_k) = \frac{1}{Z} \frac{Z}{e(1+\delta_k)} \cdot \frac{e}{Z} \cdot \log_2 e = -\frac{1}{Z} \frac{\log_2 e}{1+\delta_k} ; \ \mathcal{E}_k''(0) = \frac{1}{Z} \log_2 e .$$

If $|\delta_k| \ll 1$ (all states are *almost* equiprobable), then

$$\mathcal{E}_k(\delta_k) \approx \mathcal{E}_k(0) + \mathcal{E}_k'(0)\delta_k + \frac{1}{2}\mathcal{E}_k''(0)\delta_k^2 = \frac{1}{Z}\log_2 Z + \frac{1}{Z}\log_2(Ze^{-1})\delta_k - (\frac{1}{2Z}\log_2 e)\delta_k^2 .$$

The Shannon entropy

$$\mathcal{E} = \sum_{k=1}^{Z}\mathcal{E}_k(\delta_k) \approx \sum_{k=1}^{Z}\frac{1}{Z}\log_2 Z + \frac{1}{Z}\log_2(Ze^{-1})\sum_{k=1}^{Z}\delta_k - \frac{\log_2 e}{2Z}\sum_{k=1}^{Z}\delta_k^2 = \log_2 Z - \frac{\log_2 e}{2Z}\sum_{k=1}^{Z}\delta_k^2 .$$

Let us denote the root-mean-square deviation as $\sigma \equiv \frac{1}{Z}\sum_{k=1}^{Z}\delta_k^2$. Then

$$\boxed{\mathcal{E}(Shannon) \approx \log_2 Z - \frac{\log_2 e}{2}\sigma = I - \frac{\log_2 e}{2}\sigma,} \text{ if } |\delta_k| \ll 1 \tag{6.1}$$

and

$$\boxed{\mathcal{E} \le I}.$$

It means that the Shannon entropy of a state generator is less (or equal) than the information amount which it theoretically can produce. $\mathcal{E} = I$ is only in an ideal case, if all states of the state generator are *exactly* equiprobable, i.e. are exactly homogeneously distributed.

If some states occur very often and others – very seldomly (the states are inhomogeneously distributed), then $\mathcal{E} \to 0$. It means that entropy $\mathcal{E}$ indicates the homogeneity of state distribution of a state generator.

6.2 A2: Geometry-neutral Phase Translation Equation

Phase Translation Operator

Let $\hat{\mathrm{F}}(\Delta\Phi_j)$ be the operator of the phase translation during transition of a system from the state at the time step `(j-1)` ($|\psi\rangle$) into the next state `j` ($|\psi'\rangle$), whereby $\Delta\Phi_j \equiv \Phi_j - \Phi_{j-1}$ is the phase change at this transition. Then

$$|\psi'\rangle = \hat{\mathrm{F}}(\Delta\Phi_j)\,|\psi\rangle .$$

Since phases of neighbouring states might slightly differ from each other, we can write down

$$|\psi'\rangle \approx (1 + i\cdot\Delta\hat{\Phi}_j)\,|\psi\rangle = (1 + i\hat{\sigma}\cdot\Delta\Phi_j)\,|\psi\rangle ,$$

whereby $\hat{\sigma}$ is the dimensionless phase translation operator. Then, the phase translation operator can be represented as

$$\hat{\mathrm{F}}(\Delta\Phi_j) \approx 1 + i\hat{\sigma}\cdot\Delta\Phi_j . \tag{6.2}$$

Since the change of phase $\Delta\Phi_j$ represents the number of action quanta necessary for the related translation ($\Delta\Phi_j = \frac{\Delta S_j}{\hbar}$, whereby ΔS_j is the action being needed by the system in order to come from microstate `(j-1)` into microstate `j`), we can state

$$\hat{\mathrm{F}}(\Delta S_j) \approx 1 + \frac{i}{\hbar}\hat{\sigma}\cdot\Delta S_j \text{ and}$$

$$|\psi'\rangle \approx (1 + \frac{i}{\hbar}\hat{\sigma}\cdot\Delta S_j)\,|\psi\rangle\,. \tag{6.3}$$

General Phase Translation Equation

Let $C_j(S_j) \equiv \langle j\,|\,\psi(S_j)\rangle$ be the amplitude of being a system in the basis microstate j with action S_j. Then, the amplitude of attaining this microstate is (s. (6.3))

$C_j(S_j) = \sum_k [\delta_{(j-1)k} + \frac{i}{\hbar}\sigma_{jk}(S_j)\cdot\Delta S_j]C_k(S_k) = C_{j-1}(S_{j-1}) + \frac{i}{\hbar}\Delta S_j \sum_k \sigma_{jk}(S_j)C_k(S_k)$, whereby δ_{jk} is Kronecker's delta.

It follows from this that $\dfrac{C_j(S_j) - C_{j-1}(S_{j-1})}{\Delta S_j} \equiv \dfrac{\Delta C_j(S_j)}{\Delta S_j} = \dfrac{i}{\hbar}\sum_k \sigma_{jk}(S_j)C_k(S_k)$.

Thus, the general phase translation equation can be represented as

$$i\hbar\frac{\Delta C_j(S_j)}{\Delta S_j} = -\sum_k \sigma_{jk}(S_j)C_k(S_k)\,, \tag{6.4}$$

or dimensionless with the phase Φ_j

$$i\frac{\Delta C_j(\Phi_j)}{\Delta\Phi_j} = -\sum_k \sigma_{jk}(\Phi_j)C_k(\Phi_k)\,. \tag{6.5}$$

This *phase translation equation* does not know any coordinates and, thus, is geometry-neutral. Also, no assumptions were made about differentiability of any parameters.

Stationary States and Conservation of Observables

It is interesting to note that, for the special case of stationary systems where their observable states remain, we obtain

$|\psi_0'\rangle \equiv |\psi_0\rangle \;\Rightarrow\; |\psi_0\rangle = \hat{\mathrm{F}}(\Delta\Phi_{j+1})\,|\psi_0\rangle \;\Rightarrow\; \hat{\mathrm{F}}(\Delta\Phi_{j+1})\,|\psi_0\rangle = e^{im\Delta\Phi_{j+1}}\,|\psi_0\rangle \approx (1 + im\Delta\Phi_{j+1})\,|\psi_0\rangle$
(m is an integer).

It means that the dimensionless phase translation operator $\hat{\sigma}\,|\psi_0\rangle \approx m\,|\psi_0\rangle$, for these special states of conservation of observables, represents just an integer.

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